FREE NOTES · NCERT + JEE/NEET LEVEL

Alternating Current

AC voltage, rms value, AC with R, L and C, phasors, the LCR circuit, impedance, resonance, power factor, LC oscillations and the transformer. With phasor diagrams and solved numericals.

01 What is AC
02 RMS & average value
03 AC across a resistor
04 Series LCR circuit
05 Resonance
06 Power & power factor
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01 What is AC

A voltage or current that keeps reversing direction with time **like a sine wave**.

$$v = V_m \sin \omega t,\ i = I_m \sin(\omega t + \varphi)$$

V_m = peak value, $\omega = 2\pi f$. Household supply in India: 220 V (rms), 50 Hz.

Why AC? Transformers can easily step its voltage up or down, so it can be sent long distances at high voltage with low loss.

02 RMS & average value

$I_{rms} = I_m/\sqrt{2} \approx 0.707\ I_m$

$V_{rms} = V_m/\sqrt{2}$

Average over a full cycle $= 0$; over a half cycle $= 2I_m/\pi \approx 0.637\ I_m$

WHAT RMS MEANS

The DC value that produces **the same heating** in a resistor. Voltmeters, ammeters and the household "220 V" all give rms values. Peak = 220√2 ≈ 311 V.

03 AC across a resistor

v and i are **in phase** (φ = 0). $I_m = V_m/R$. Power $P = I^2_{rms}R$.

04 AC across an inductor

Inductive reactance $X_L = \omega L = 2\pi fL$

Current **lags** voltage by π/2. Higher f ⇒ larger X_L. For DC (f = 0), X_L = 0. Average power = 0.

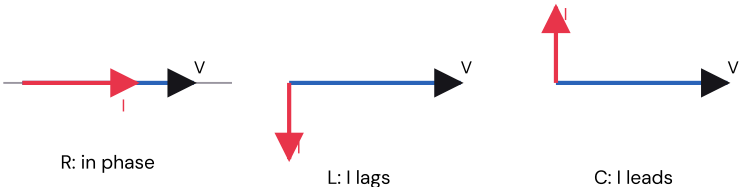
05 AC across a capacitor

Capacitive reactance $X_C = 1/(\omega C) = 1/(2\pi fC)$

Current **leads** voltage by π/2. Higher f ⇒ smaller X_C. For DC, $X_C = \infty$ (a capacitor blocks DC). Average power = 0.

MEMORY TRICK: "CIVIL"

In **C**, **I** comes before **V** (current leads); **V** comes before **I** in **L** (voltage leads, current lags).



Phasors rotate anticlockwise; leading = ahead (up)

04 Series LCR circuit

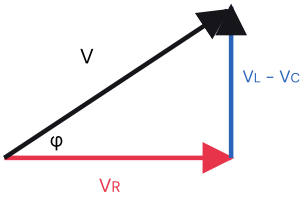
$$Z = \sqrt{(R^2 + (X_L - X_C)^2)}$$

$$I = V/Z$$

$$\tan \varphi = (X_L - X_C)/R$$

Z = impedance (Ω). $X_L > X_C$: inductive (I lags). $X_C > X_L$: capacitive (I leads).

Voltages: $V = \sqrt{(V_R^2 + (V_L - V_C)^2)}$. V_L or V_C alone can exceed the supply voltage!



Voltage triangle (like the impedance triangle)

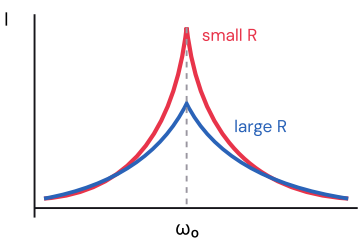
05 Resonance

When $X_L = X_C$: Z = R (minimum), current is **maximum**, φ = 0.

$$\omega_0 = 1/\sqrt{(LC)},\ f_0 = 1/(2\pi\sqrt{(LC)})$$

Quality factor $Q = \omega_0 L/R = (1/R)\sqrt{(L/C)}$

Higher Q ⇒ sharper peak ⇒ sharper tuning. Bandwidth = R/L = ω_0/Q.



Smaller R ⇒ taller, narrower peak

Radio/TV tuning: vary the capacitor to bring the circuit into resonance with a station's frequency. Resonance needs **both L and C**.

06 Power & power factor

$$P_{avg} = V_{rms}\, I_{rms} \cos\varphi$$

Power factor $\cos\varphi = R/Z$

Circuit	φ	cos φ	Power
Only R / at resonance	0	1	Maximum ($V_{rms}I_{rms}$)
Only L or only C	π/2	0	0 (**wattless current**)

WATTLESS CURRENT

In pure L or C current flows but average power is 0: energy is stored in one half-cycle and returned in the next. That's why a **choke coil** (large L, small R) reduces current better than a resistor (no energy wasted). Factories add capacitors to improve a low power factor.

07 LC oscillations

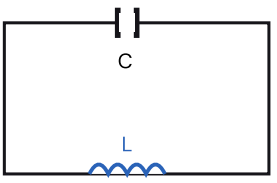
Connect a charged capacitor to an inductor: energy passes from the capacitor's electric field to the inductor's magnetic field and back, again and again.

$$\omega = 1/\sqrt{(LC)}$$

$q = q_0 \cos \omega t$

Total energy $= q_0^2/(2C) = \frac{1}{2}LI_0^2$

Like spring-mass SHM: L ↔ m, 1/C ↔ k, q ↔ x. With R, the oscillations are damped.



LC circuit: energy swings back and forth

08 Transformer

Based on mutual induction: AC in the primary ⇒ changing flux in the core ⇒ EMF in the secondary. Works **only with AC**.

$$V_s/V_p = N_s/N_p = I_p/I_s$$

Ideal: $V_pI_p = V_sI_s$. Step-up: $N_s > N_p$ (voltage up, current down). Step-down: the reverse. Efficiency = output/input.

Energy loss	How to reduce
Copper loss (I^2R)	Thick wires
Eddy currents	Laminated core
Hysteresis	Soft iron core
Flux leakage	Wind the coils one over the other

09 JEE Extra + Tricks

JEE LEVEL

- Phase trick "**ELI the ICE man**": in L, voltage (E) leads current (I) by 90°; in C, current leads voltage by 90°.
- Impedance $Z = \sqrt{(R^2 + (X_L - X_C)^2)}$; $\tan\varphi = (X_L - X_C)/R$; resonance $\omega_o = 1/\sqrt{(LC)}$, $Z = R$, current max.
- Quality factor $Q = \omega_o L/R = (1/R)\sqrt{(L/C)}$; bandwidth $= R/L$ (in ω).
- Average power $P = V_{rms} I_{rms} \cos\varphi$; pure L or C $\Rightarrow \cos\varphi = 0$ (wattless current).
- $I_{rms} = I_o/\sqrt{2}$ for sine wave; average of sine current over full cycle = 0, over half cycle $= 2I_o/\pi$.
- Ideal transformer: $V_s/V_p = N_s/N_p = I_p/I_s$.

10 Solved numericals

Q1. Peak voltage of a 220 V rms supply?
$V_m = 220\sqrt{2} \approx 311$ V. **Ans: ≈ 311 V**

Q2. A 0.5 H inductor at 50 Hz. X_L? (π = 3.14)
$X_L = 2\pi \times 50 \times 0.5 = 157\ \Omega$. **Ans: 157 Ω**

Q3. LCR: R = 30 Ω, X_L = 80 Ω, X_C = 40 Ω, V = 200 V. Find Z, I, power factor and power.
$Z = \sqrt{(900 + 1600)} = 50\ \Omega$. I = 4 A. $\cos\varphi = 30/50 = 0.6$. $P = 200 \times 4 \times 0.6 = 480$ W. **Ans: 50 Ω, 4 A, 0.6, 480 W**

Q4. L = 1 mH, C = 10 µF. Resonant frequency?
$\omega_o = 1/\sqrt{(10^{-3} \times 10^{-5})} = 1/\sqrt{(10^{-8})} = 10^4$ rad/s ⇒ $f_o \approx 1592$ Hz. **Ans: ≈ 1.6 kHz**

Q5. A transformer steps 220 V down to 11 V with 1000 primary turns. Secondary turns? If the secondary carries 2 A, primary current (ideal)?
$N_s = 1000 \times 11/220 = 50$. $I_p = 2 \times 11/220 = 0.1$ A. **Ans: 50, 0.1 A**

11 Common mistakes

- Taking the full-cycle average of AC as $2I_m/\pi$ (it's 0 over a full cycle).
- Mixing up lead and lag for L and C (CIVIL).
- Adding $Z = R + X_L + X_C$ directly (add as vectors).
- Thinking wattless current means no current (current flows, power is 0).
- Running a transformer on DC or thinking it creates energy.

12 Formula sheet

RMS	$V_{rms} = V_m/\sqrt{2}$
Reactances	$X_L = \omega L,\ X_C = 1/\omega C$
Impedance	$Z = \sqrt{(R^2 + (X_L - X_C)^2)}$
Phase	$\tan\varphi = (X_L - X_C)/R$
Resonance	$\omega_0 = 1/\sqrt{(LC)}$
Q factor	$\omega_0 L/R$
Power	$P = V_{rms} I_{rms} \cos\varphi,\ \cos\varphi = R/Z$
LC oscillation	$\omega = 1/\sqrt{(LC)}$
Transformer	$V_s/V_p = N_s/N_p = I_p/I_s$

13 Practice questions

1. Frequency is doubled. X_L and X_C?
2. Power factor of an LCR circuit at resonance?
3. L and C are both made 4 times. Resonant frequency?
4. What does a capacitor do with DC?

ANSWERS

1) X_L doubles, X_C halves · 2) 1 · 3) 1/4 · 4) Once charged, it blocks current ($X_C = \infty$)

14 Quick revision: 5 minutes before the exam

BASICS

- $V_{rms} = V_m/\sqrt{2}$.
- R: in phase; L: I lags; C: I leads.
- $X_L = \omega L$; $X_C = 1/\omega C$.

LCR

- $Z = \sqrt{(R^2 + (X_L - X_C)^2)}$.
- Resonance: $X_L = X_C$, Z = R, I maximum.
- $\omega_o = 1/\sqrt{(LC)}$.

POWER

- $P = VI \cos \varphi$; $\cos \varphi = R/Z$.
- Pure L/C: wattless.
- Choke coil: no energy loss.

TRANSFORMER

- $V \propto N$, $I \propto 1/N$.
- Laminated soft iron core.
- AC only.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.