FREE NOTES · NCERT + JEE/NEET LEVEL

Electrostatic Potential & Capacitance

Potential, potential energy, equipotential surfaces, conductors, dielectrics, capacitors, series-parallel combinations and stored energy. A formula box and solved numericals with every topic.

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01 Electric potential

The work needed to bring a unit positive charge **from infinity to a point** (without acceleration) is the potential at that point.

$$V = W / q$$

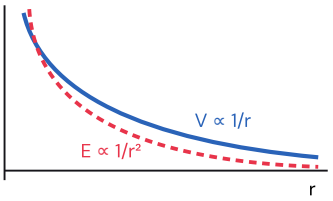
Unit volt (V) = J/C. A scalar. Potential difference: $V_B - V_A = W_{AB}/q$. A positive charge moves from high to low potential (like water flowing downhill).

The electrostatic force is conservative, so work depends only on the start and end points, not the path.

02 Potential of a point charge

$$V = k\,q / r$$

Positive for +q, negative for −q. $V \propto 1/r$ (field $\propto 1/r^2$). V = 0 at infinity.



Potential falls slowly, field falls fast

Charged conducting sphere (R, q): outside $V = kq/r$; on the surface and everywhere inside $V = kq/R$ (constant).

03 Potential of a dipole and a system of charges

DIPOLE (R ≫ A)

$$V = k\,p\cos\theta\,/\,r^2$$

Axial line (θ = 0): kp/r². Equatorial line (θ = 90°): **V = 0** (but the field is not 0!). $V \propto 1/r^2$.

MANY CHARGES

$$V = k\,(q_1/r_1 + q_2/r_2 + \ldots)$$

A plain algebraic sum (scalar); no directions to worry about. Add with signs.

EXAMPLE

+q at all four corners of a square, at the centre: V = 4kq/(a/√2), field = 0. With +q, −q, +q, −q: V = 0 and field = 0 too.

04 Equipotential surfaces

A surface where every point has the same potential. Moving a charge on it needs **zero work**.

- Field lines are always **perpendicular** to equipotentials.
- Two equipotentials never intersect.
- Point charge: concentric spheres. Uniform field: planes perpendicular to the field.
- Where equipotentials are close together, the field is strong.
- A conductor's surface is an equipotential.

05 Relation between E and V

$$E = -dV/dr$$

The field points in the direction in which potential falls fastest. Uniform field: $E = V/d$. Hence E can also be in V/m.

NOTE

V = 0 does not mean E = 0 (equatorial point of a dipole). E = 0 does not mean V = 0 (inside a charged shell V is constant and non-zero).

06 Potential energy

TWO CHARGES

$$U = k\, q_1\, q_2 / r$$

Like signs: U positive (repulsion; work needed to bring them close). Unlike signs: negative (bound).

THREE OR MORE

$$U = k(q_1q_2/r_{12} + q_2q_3/r_{23} + q_1q_3/r_{13})$$

Each pair once.

PE of a charge q in an external field: $U = qV$. For a dipole: $U = -pE\cos\theta$.

ELECTRON VOLT

Energy gained by an electron through 1 V: $1\ eV = 1.6 \times 10^{-19}$ J. To find speed: $qV = \frac{1}{2}mv^2$.

07 Conductors (electrostatics)

Property	Why
E = 0 inside	Free electrons move until the field is cancelled
E at the surface is perpendicular to it	A parallel part would make charge flow
All charge lies on the surface	By Gauss's law, net charge inside is 0
The whole conductor is equipotential	E = 0 ⇒ V constant
Charge density is higher at sharp points	σ ∝ 1/R (more curvature ⇒ higher σ)

At a sharp point σ is so large that the field ionises the air: **corona discharge**. A **lightning conductor** works on this.

08 Dielectrics & polarisation

A **dielectric** is an insulator that becomes **polarised** in an electric field (its charges shift slightly, forming tiny dipoles).

Type	Without a field	Example
Polar	Molecules already have dipole moments (random)	H_2O, HCl
Non-polar	Zero dipole moment; induced by the field	O_2, N_2, CO_2

The field inside is reduced: $E = E_0/K$

Dielectric constant $K = \varepsilon/\varepsilon_0 = \varepsilon_r$ (vacuum $= 1$, metal $= \infty$)

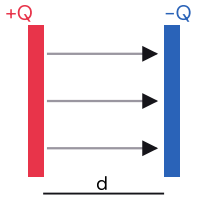
Polarisation $P = \chi_e E$, $K = 1 + \chi$ (χ = susceptibility).

09 Capacitor

Two conductors (plates) with an insulator between them. It stores charge and energy.

$$C = Q / V$$

Unit farad (F) = C/V. 1 F is very large; μF and pF are common. C depends only on **shape, size and the medium between**, not on Q or V.



Uniform field between the plates $E = \sigma/\varepsilon_o$

PARALLEL PLATE CAPACITOR

$$C = \varepsilon_0 A / d$$

Larger area ⇒ larger C; larger gap ⇒ smaller C. Isolated sphere: $C = 4\pi\varepsilon_0 R$ (Earth ≈ 711 μF).

10 Capacitor with a dielectric

Completely filled: $C = K\,\varepsilon_0\,A / d = K\,C_0$

Slab of thickness t: $C = \varepsilon_0 A / (d - t + t/K)$

Metal slab ($K = \infty$): $C = \varepsilon_0 A/(d - t)$

Dielectric inserted with	Q	V	E	C	Energy U
Battery connected (V fixed)	× K	Same	Same	× K	× K
Battery removed (Q fixed)	Same	÷ K	÷ K	× K	÷ K

11 Series & parallel

SERIES

$$1/C = 1/C_1 + 1/C_2 + \ldots$$

Same charge on each capacitor. Voltage divides: more V across the smaller C. Total C is less than the smallest.

PARALLEL

$$C = C_1 + C_2 + \ldots$$

Same V across each. Charge divides: more Q on the larger C.

REMEMBER

Capacitor combinations are the **opposite** of resistors: reciprocals add in series, values add in parallel. n equal capacitors: series C/n, parallel nC.

12 Stored energy

$$U = \tfrac{1}{2} C V^2 = Q^2/(2C) = \tfrac{1}{2} Q V$$

Energy density $u = \tfrac{1}{2} \varepsilon_0 E^2$ (J/m³)

The energy is stored in the field between the plates.

CHARGE SHARING (JOINING TWO CAPACITORS)

Common potential: $V = (C_1V_1 + C_2V_2)/(C_1 + C_2)$. Energy loss: $\Delta U = C_1C_2(V_1 - V_2)^2/(2(C_1 + C_2))$. The energy goes to heat and radiation; there is always a loss.

Charging a capacitor from a battery: the battery supplies QV, the capacitor stores ½QV, and half becomes heat.

13 JEE Extra + Tricks

JEE LEVEL

- Capacitor with dielectric slab of thickness t: $C = \varepsilon_0 A/(d - t + t/K)$; with metal slab: $C = \varepsilon_0 A/(d - t)$.
- Energy lost when two charged capacitors are joined: $\Delta U = \frac{1}{2}\cdot C_1C_2/(C_1 + C_2)\cdot(V_1 - V_2)^2$ (appears as heat).
- Force between plates $F = Q^2/(2\varepsilon_0 A)$; energy density $u = \frac{1}{2}\varepsilon_0 E^2$.
- Battery connected (V constant) vs disconnected (Q constant): inserting dielectric ⇒ C × K; if V constant then Q, U × K; if Q constant then V, U ÷ K.
- Potential of a conducting sphere is the same everywhere inside = kQ/R.

14 Solved numericals ($k = 9 \times 10^9$)

Q1. Potential 9 cm from a 4 µC charge?
$V = 9 \times 10^9 \times 4 \times 10^{-6}/0.09 = 4 \times 10^5$ V. **Ans: 4×10^5 V**

Q2. Work to move 2 µC from a point at 100 V to one at 300 V?
$W = q(V_B - V_A) = 2 \times 10^{-6} \times 200 = 4 \times 10^{-4}$ J. **Ans: 4×10^{-4} J**

Q3. 2 µC and −3 µC are 10 cm apart. PE of the system?
$U = 9 \times 10^9 \times (-6 \times 10^{-12})/0.1 = -0.54$ J. **Ans: −0.54 J**

Q4. Parallel plates: A = 100 cm², d = 1 mm. C? ($\varepsilon_0 = 8.85 \times 10^{-12}$)
$C = 8.85 \times 10^{-12} \times 0.01/0.001 = 8.85 \times 10^{-11}$ F ≈ 88.5 pF. **Ans: ≈ 88.5 pF**

Q5. 2, 3 and 6 µF in series across 12 V. Total C and charge on each?
1/C = 1/2 + 1/3 + 1/6 = 1 ⇒ C = 1 µF. Q = 12 µC (same on all). **Ans: 1 µF, 12 µC**

Q6. A 10 µF capacitor is charged to 200 V. Energy?
$U = \frac{1}{2} \times 10^{-5} \times 4 \times 10^4 = 0.2$ J. **Ans: 0.2 J**

Q7. A 4 µF capacitor at 50 V is joined to an uncharged 6 µF. Common V and energy loss?
$V = 4 \times 50/10 = 20$ V. Before $\frac{1}{2} \times 4 \times 10^{-6} \times 2500 = 5$ mJ, after $\frac{1}{2} \times 10 \times 10^{-6} \times 400 = 2$ mJ ⇒ loss 3 mJ.
Ans: 20 V, 3 mJ

15 Common mistakes

- Adding potentials as vectors. V is a scalar; add with signs.
- Combining capacitors like resistors (it's the opposite).
- Answering dielectric questions without checking whether the battery is connected.
- Assuming V = 0 ⇒ E = 0.
- Thinking the potential inside a conductor is 0 (it's constant, equal to the surface value).
- Mixing up pF = 10^{-12} and μF = 10^{-6}.

16 Formula sheet

Potential	$V = W/q,\ V = kq/r$
Dipole	$V = kp\cos\theta/r^2$
Field from V	$E = -dV/dr,\ E = V/d$
PE	$U = kq_1q_2/r,\ U = qV$
Capacitance	$C = Q/V$
Parallel plate	$C = K\varepsilon_0 A/d$
Partial slab	$C = \varepsilon_0 A/(d - t + t/K)$
Sphere	$C = 4\pi\varepsilon_0 R$
Series / parallel	$1/C = \Sigma 1/C_i\ /\ C = \Sigma C_i$
Energy	$\frac{1}{2}CV^2 = Q^2/2C,\ u = \frac{1}{2}\varepsilon_0 E^2$
Common potential	$(C_1V_1 + C_2V_2)/(C_1 + C_2)$

17 Practice questions

1. The plate gap of a parallel plate capacitor is doubled and the area halved. C?
2. The battery is removed and a K = 5 dielectric inserted. V and U?
3. Three 3 μF capacitors: C in series and in parallel?
4. An electron is accelerated through 100 V. Energy (eV and J)?
5. V and E on the equatorial line of a dipole?

ANSWERS

1) C/4 · 2) V/5, U/5 · 3) 1 μF, 9 μF · 4) 100 eV = 1.6×10^{-17} J · 5) V = 0, E = kp/r^3 (not 0)

18 Quick revision: 5 minutes before the exam

POTENTIAL

- $V = kq/r$ (scalar).
- Dipole equatorial: $V = 0$.
- $E = -dV/dr$; $E \perp$ equipotential.
- Conductor: $E = 0$ inside, V constant.

ENERGY

- $U = kq_1q_2/r$; every pair.
- $1\ eV = 1.6 \times 10^{-19}$ J.
- Capacitor $\frac{1}{2}CV^2$, $u = \frac{1}{2}\varepsilon_0 E^2$.

CAPACITOR

- $C = K\varepsilon_0 A/d$.
- Series: Q same; parallel: V same.
- Battery on: V fixed; off: Q fixed.

DIELECTRIC

- E inside $= E_0/K$.
- Polar (H_2O) vs non-polar (O_2).
- Energy is lost in charge sharing.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.