FREE NOTES · NCERT + JEE/NEET LEVEL

Electric Charges & Fields

Properties of charge, Coulomb's law, electric field, field lines, the dipole, electric flux and all the applications of Gauss's law. With diagrams and solved numericals.

01 Electric charge
02 Properties of charge
03 Methods of charging
04 Coulomb's law
05 Superposition principle
06 Electric field
07 Electric field lines
08 Electric dipole
09 Dipole in a uniform field
10 Electric flux
11 Gauss's law
12 Applications of Gauss's law
13 JEE Extra + Tricks
14 Solved numericals ($k = 9 \times 10^9$)
15 Common mistakes
16 Formula sheet
17 Practice questions
18 Quick revision

01 Electric charge

A basic property of matter that makes it feel and exert electric forces. Two kinds: **positive** and **negative** (named by Benjamin Franklin). **Like charges repel, unlike charges attract.**

Rub a glass rod with silk: rod +, silk −. Rub a plastic rod with fur: rod −, fur +. Electrons move from one to the other; protons don't move.

CONDUCTOR VS INSULATOR

Conductors (metals, the human body, the Earth): charge flows easily, free electrons. **Insulators** (glass, plastic, rubber): charge stays where it is placed. **Earthing**: connecting a conductor to the ground, so extra charge flows into the Earth.

02 Properties of charge

Property	Meaning
Additivity	Total charge = algebraic sum of all charges (scalar). +2, −5, +1 ⇒ −2.
Conservation	The total charge of an isolated system is neither created nor destroyed, only transferred.
Quantization	$q = n\,e,\ e = 1.6 \times 10^{-19}\ C,\ n$ an integer

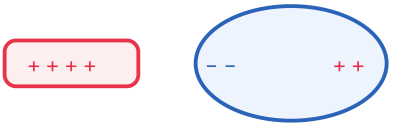
SI unit: coulomb (C). 1 C ≈ 6.25 × 10^{18} electrons. At the macroscopic level quantization isn't noticeable because e is so small.

03 Methods of charging

Method	How	Sign of charge
Friction (rubbing)	Rub two objects, electrons transfer	Equal and opposite on the two
Conduction (contact)	Touch with a charged body	Same sign
Induction (no contact)	Bring a charged rod near, earth the far end, remove the earth, then remove the rod	Opposite sign

WHY DOES LIGHT PAPER FLY TO THE ROD?

The charged rod induces an opposite charge on the near side of the paper. Opposite charge is closer, like charge farther ⇒ net attraction ⇒ the paper sticks. **A charged body can attract even a neutral one.**



Induction: opposite charge on the near side

04 Coulomb's law

$$F = k\, q_1\, q_2 / r^2 = q_1 q_2 / (4\pi\varepsilon_0\, r^2)$$

$k = 9 \times 10^9$ N·m²/C², $\varepsilon_0 = 8.854 \times 10^{-12}$ C²/N·m² (permittivity of free space). The force acts along the line joining the charges. For point charges.

IN A MEDIUM

In a medium of dielectric constant $K = \varepsilon_r$ the force is K times smaller: $F_{medium} = F_{vacuum}/K$. Water has $K \approx 80$, which is why salt dissolves in it (Na^+ Cl^- attraction falls 80 times).

COMPARED WITH GRAVITY

Between two protons the electric force is ~10^{36} times the gravitational force. But electric force can attract or repel; gravity only attracts.

05 Superposition principle

With many charges, the net force on one charge = **vector sum** of the forces from each other charge. Each pair's force is unaffected by the others.

$$F_1 = F_{12} + F_{13} + \ldots + F_{1n} \text{ (vector)}$$

EQUILIBRIUM TRICK

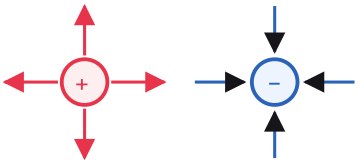
Where to put a third charge between two like charges so the net force on it is 0? Nearer the weaker one: $x/(d-x) = \sqrt{(q_1/q_2)}$. For unlike charges this point lies outside, beyond the smaller charge.

06 Electric field

A charge creates a "region of influence" around itself. The force on a **unit positive test charge** at a point = electric field.

$$E = F/q_0,\ E = kq/r^2$$

Unit N/C = V/m. A vector. Points away from a positive charge, toward a negative one. Force on charge q: $F = qE$.



Outward from +, inward to −

Continuous charge: linear density λ (C/m), surface density σ (C/m²), volume density ρ (C/m³). Find the field of a small piece dq and integrate.

07 Electric field lines

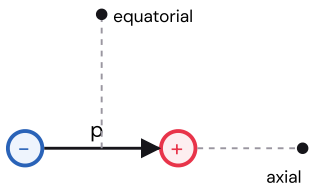
- Start on positive and end on negative charges (or go to infinity for a single charge).
- The tangent at a point gives the field direction.
- **Two lines never cross** (otherwise one point would have two directions).
- Where lines are crowded the field is strong.
- They don't form closed loops (in an electrostatic field).
- They meet a conductor's surface at 90°; the field inside a conductor is 0.

08 Electric dipole

Two equal and opposite charges (+q, −q) a small distance 2a apart. Examples: H_2O, HCl molecules.

$$p = q \times 2a$$

Dipole moment, a vector pointing **from −q to +q**. Unit C·m.



Axial line vs equatorial line

Point	Field (r ≫ a)	Direction
Axial line	$E = 2kp/r^3$	Along p
Equatorial line	$E = kp/r^3$	Opposite to p

REMEMBER

A dipole's field falls as $1/r^3$ (a point charge's as $1/r^2$), so it dies off quickly. $E_{axial} = 2 \times E_{equatorial}$ (same r).

09 Dipole in a uniform field

Net force $= 0$ (equal and opposite on the two)

$$\text{Torque } \tau = p \times E, \quad |\tau| = pE \sin\theta$$

Potential energy $U = -p \cdot E = -pE\cos\theta$

θ	Torque	U	Equilibrium
0° (p ∥ E)	0	-pE (min)	Stable
90°	pE (max)	0	—
180°	0	+pE (max)	Unstable

Work to rotate from θ_1 to θ_2 = $pE(\cos\theta_1 - \cos\theta_2)$. In a non-uniform field there is also a net force.

10 Electric flux

A measure of how many field lines pass through a surface.

$$\Phi = E \cdot A = E\,A\cos\theta$$

θ = angle between E and the area vector (normal to the surface). Unit N·m²/C. A scalar. For a closed surface, outgoing lines count +, incoming −.

11 Gauss's law

STATEMENT

$\oint E \cdot dA = q_{enclosed} / \varepsilon_o$

Total flux through any closed surface (Gaussian surface) = enclosed charge / ε_o. Charges outside give zero net flux (though they do affect the field).

- The shape or size of the surface doesn't matter, only the charge inside.
- Charge q at the centre of a cube: flux through each face = $q/(6\varepsilon_o)$. Charge at a corner: total = $q/(8\varepsilon_o)$.
- Choose a Gaussian surface where E is constant or ⊥/∥ everywhere (symmetry).

12 Applications of Gauss's law

Charge distribution	Field	Depends on
Infinite line (λ)	$E = \lambda/(2\pi\varepsilon_0 r)$	$\propto 1/r$
Infinite plane sheet (σ)	$E = \sigma/(2\varepsilon_0)$	Independent of distance
Between two opposite sheets (+σ, −σ)	$E = \sigma/\varepsilon_0$ (0 outside)	Capacitor
Just outside a conductor	$E = \sigma/\varepsilon_0$	Perpendicular to surface
Spherical shell (R, q): outside	$E = kq/r^2$	Like a point charge
Spherical shell: inside	E = 0	—
Solid sphere (uniform ρ): inside	$E = kqr/R^3$	$\propto r$

ELECTROSTATIC SHIELDING

The field inside a hollow conductor is always 0. That's why sitting in a car during lightning is safe (the car is a metal box = Faraday cage).

13 JEE Extra + Tricks

JEE LEVEL

- Field on axis of a ring: $E = kQx/(x^2 + R^2)^{3/2}$; maximum at $x = R/\sqrt{2}$; zero at centre.
- Dipole: axial $E = 2kp/r^3$, equatorial $E = kp/r^3$ (opposite to p); torque $\tau = p \times E$; PE $U = -p \cdot E$.
- Flux tricks: charge at centre of cube ⇒ each face $q/6\varepsilon_0$; charge at a corner ⇒ $q/(24\varepsilon_0)$ through the three far faces only.
- Field due to infinite line $\lambda/(2\pi\varepsilon_0 r)$; infinite sheet $\sigma/(2\varepsilon_0)$; just outside a conductor σ/ε_0.
- Two charges: null point lies outside the pair (near smaller charge) for unlike charges, between them for like charges.

14 Solved numericals ($k = 9 \times 10^9$)

Q1. A body has charge -3.2×10^{-7} C. How many extra electrons?
$n = 3.2 \times 10^{-7}/1.6 \times 10^{-19} = 2 \times 10^{12}$. **Ans: 2×10^{12} electrons**

Q2. 2 µC and 3 µC are 30 cm apart. Force?
$F = 9 \times 10^9 \times 6 \times 10^{-12}/0.09 = 0.6$ N (repulsive). **Ans: 0.6 N**

Q3. 4 µC and 9 µC are 1 m apart. Where should a third charge be placed so the net force on it is 0?
$x/(1 - x) = \sqrt{(4/9)} = 2/3$ ⇒ $x = 0.4$ m (from 4 µC). **Ans: 40 cm from 4 µC**

Q4. Field 10 cm from a 5 µC charge?
$E = 9 \times 10^9 \times 5 \times 10^{-6}/0.01 = 4.5 \times 10^6$ N/C. **Ans: 4.5×10^6 N/C**

Q5. A dipole ($p = 4 \times 10^{-9}$ C·m) is at 30° to a 5×10^4 N/C field. Torque?
$\tau = pE \sin 30° = 4 \times 10^{-9} \times 5 \times 10^4 \times 0.5 = 10^{-4}$ N·m. **Ans: 10^{-4} N·m**

Q6. A 10 cm cube has 8.85 µC at its centre. Flux through one face? ($\varepsilon_0 = 8.85 \times 10^{-12}$)
Total $= q/\varepsilon_0 = 10^6$. One face $= 10^6/6 \approx 1.67 \times 10^5$ N·m²/C. **Ans: ≈ 1.67×10^5 N·m²/C**

Q7. Field 2 cm from an infinite line ($\lambda = 2 \times 10^{-7}$ C/m)?
$E = 2k\lambda/r = 2 \times 9 \times 10^9 \times 2 \times 10^{-7}/0.02 = 1.8 \times 10^5$ N/C. **Ans: 1.8×10^5 N/C**

15 Common mistakes

- Not converting μC to 10^{-6} or cm to m.
- Taking the dipole moment from + to - (correct: - to +).
- Counting outside charges in the flux.
- Thinking an infinite sheet's field depends on distance.
- Using kq/r^2 inside a shell (it's 0 inside).
- Assuming a net force on a dipole in a uniform field (only torque).

16 Formula sheet

Quantization	$q = ne,\ e = 1.6 \times 10^{-19}\ C$
Coulomb's law	$F = kq_1q_2/r^2,\ k = 1/4\pi\varepsilon_0 = 9 \times 10^9$
In a medium	F/K
Field	$E = F/q = kq/r^2$
Dipole moment	$p = q(2a)$
Dipole field	axial $2kp/r^3$, equatorial kp/r^3
Torque, PE	$\tau = pE \sin\theta,\ U = -pE\cos\theta$
Flux	$\Phi = EA\cos\theta$
Gauss	$\oint E\cdot dA = q/\varepsilon_0$
Line, sheet, conductor	$\lambda/2\pi\varepsilon_0 r,\ \sigma/2\varepsilon_0,\ \sigma/\varepsilon_0$
Shell	kq/r^2 outside, 0 inside

17 Practice questions

1. The distance between two charges is halved and both charges doubled. Force?
2. What is the same force in water (K = 80)?
3. Work to rotate a dipole in a uniform field from 0° to 180°?
4. A closed surface encloses +3 μC and −5 μC. Net flux?
5. Two sheets +σ and −σ. Field between them and outside?

ANSWERS

1) 4 × 4 = 16 times · 2) F/80 · 3) 2pE · 4) −2 μC/ε_0 (inward) · 5) σ/ε_0 between, 0 outside

18 Quick revision: 5 minutes before the exam

CHARGE & FORCE

- $q = ne$; conserved; additive.
- $F = kq_1q_2/r^2$; divide by K in a medium.
- Induction: opposite charge.

FIELD

- $E = kq/r^2$, $F = qE$.
- Lines: + to −, never cross.
- $E = 0$ inside a conductor.

DIPOLE

- $p = q \cdot 2a$, − to +.
- Axial $2kp/r^3$, equatorial kp/r^3.
- $\tau = pE \sin\theta$, $U = -pE\cos\theta$.

GAUSS

- $\Phi = q_{in}/\varepsilon_o$.
- Line $\lambda/2\pi\varepsilon_o r$; sheet $\sigma/2\varepsilon_o$.
- Inside a shell: 0.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.