FREE NOTES · NCERT + JEE LEVEL

Probability

Conditional probability, multiplication theorem, independent events, total probability ka theorem, Bayes' theorem, random variables aur unke probability distributions, aur random variable ka mean (expectation).

01 Conditional probability
02 Multiplication theorem
03 Independent events
04 Total probability ka theorem
05 Bayes' theorem
06 Random variables & distributions
07 Mean (expectation)
08 JEE Extra + Tricks
09 Solved examples
10 Common mistakes
11 Practice questions
12 Quick revision: exam se 5 minute pehle

01 Conditional probability

$$P(E|F) = P(E \cap F)/P(F),\ P(F) \neq 0$$

F ho chuka ho to E ki probability. Properties: P(S|F) = P(F|F) = 1; P(E′|F) = 1 − P(E|F); P(A ∪ B|F) = P(A|F) + P(B|F) − P(A ∩ B|F).

02 Multiplication theorem

$$P(E \cap F) = P(E)\cdot P(F|E) = P(F)\cdot P(E|F)$$

$$P(E \cap F \cap G) = P(E)\cdot P(F|E)\cdot P(G|E \cap F)$$

Bina wapas rakhe nikalne me: P(do ikke) = 4/52 × 3/51.

03 Independent events

$$E, F \text{ independent} \iff P(E \cap F) = P(E) \cdot P(F)$$

Tab P(E|F) = P(E). E, F independent ho to E′ aur F, E aur F′, E′ aur F′ bhi. P(A, B me se kam se kam ek) = 1 − P(A′)P(B′).

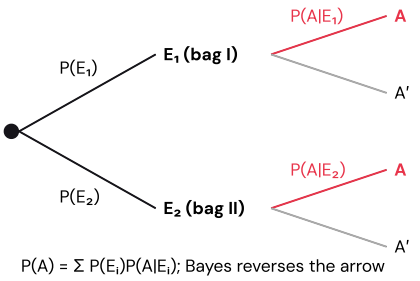
CONFUSE MAT KARO

Mutually exclusive: P(E ∩ F) = 0 (saath nahi ho sakte). Independent: ek doosre pe asar nahi. Non-zero probability wale do events dono nahi ho sakte.

04 Total probability ka theorem

$E_1, \ldots, E_n$, S ka partition hon (mutually exclusive, exhaustive, non-zero), to kisi bhi event A ke liye:

$$P(A) = \Sigma\, P(E_i) \cdot P(A|E_i)$$



Total probability aur Bayes ka tree diagram

05 Bayes' theorem

$$P(E_i|A) = P(E_i)P(A|E_i) \,/\, \Sigma_j\, P(E_j)P(A|E_j)$$

"Nateeja A mila, to kis kaaran E_i se?" $P(E_i)$ = prior, $P(E_i|A)$ = posterior.

06 Random variables & distributions

Random variable X har outcome ko ek real number deta (jaise heads ki ginti). Uska **probability distribution** har value x_i ke saath $P(x_i)$ batata, jahan har $p_i \geq 0$ aur $\Sigma p_i = 1$.

X (2 tosses me heads)	0	1	2
P(X)	1/4	1/2	1/4

07 Mean (expectation)

$$E(X) = \mu = \Sigma\, x_i\, p_i$$

Upar: E(X) = 0 + ½ + ½ = 1. (JEE extra: $Var(X) = \Sigma x_i^2 p_i - \mu^2$; binomial $P(X = r) = {}^nC_r\, p^r q^{n-r}$, mean np, variance npq.)

08 JEE Extra + Tricks

JEE LEVEL

- Bayes: $P(E_i|A) = P(E_i)P(A|E_i)/\Sigma P(E_j)P(A|E_j)$ (tree diagram madad karta).
- Independent $\Rightarrow P(A \cap B) = P(A)P(B)$; mutually exclusive $\Rightarrow P(A \cap B) = 0$ ($P(A), P(B) > 0$ ho to dono ek saath nahi).
- Binomial distribution (JEE): $P(X = r) = {}^nC_r\, p^r\, q^{n-r}$; mean np, variance npq.
- Random variable: mean $\Sigma x \cdot p$, variance $\Sigma x^2 p - (\text{mean})^2$.
- Trick: "n trials me kam se kam ek success" = $1 - q^n$.

09 Solved examples

Q1. $P(A) = 0.8$, $P(B) = 0.5$, $P(B|A) = 0.4$. $P(A \cap B)$ aur $P(A|B)$?

$P(A \cap B) = 0.8 \times 0.4 = 0.32$; $P(A|B) = 0.32/0.5 =$ **0.64**.

Q2. Dice do baar. Jod 6 hai, to P(ek 4 ho)?

Jod 6: (1,5),(2,4),(3,3),(4,2),(5,1) $\Rightarrow$ 2 me 4 $\Rightarrow$ **2/5**.

Q3. Bag I: 3 laal, 4 kaale; bag II: 5 laal, 6 kaale. Random bag se laal ball nikli. P(bag II se)?

$P = (½ \times 5/11) / (½ \times 3/7 + ½ \times 5/11) = (5/11)/(3/7 + 5/11) = 35/68$.

Q4. A aur B independent, $P(A) = 0.3$, $P(B) = 0.4$. P(dono nahi)?

$0.7 \times 0.6 =$ **0.42**.

10 Common mistakes

- $P(A|B) = P(A \cap B)/P(A)$ likhna (denominator $P(B)$ hai).
- Mutually exclusive events ko independent maan lena.
- Bayes me prior $P(E_i)$ bhool jaana.
- Distribution jiski probabilities ka jod 1 na ho.

11 Practice questions

1. $P(A) = 1/2$, $P(B) = 1/3$, A, B independent ho to $P(A \cup B)$.
2. Bina wapas rakhe do patte. P(dono king)?
3. k nikalo: $P(X = 0, 1, 2) = k, 2k, 3k$.
4. Fair dice pe number ka mean.

ANSWERS

1) 2/3 · 2) 1/221 · 3) 1/6 · 4) 3.5

12 Quick revision: exam se 5 minute pehle

CONDITIONAL

- $P(E|F) = P(E \cap F)/P(F)$.
- $P(E \cap F) = P(E)P(F|E)$.

INDEPENDENT

- $P(E \cap F) = P(E)P(F)$.
- Exclusive se alag.

TOTAL & BAYES

- $P(A) = \Sigma P(E_i)P(A|E_i)$.
- $P(E_i|A)$ = term/total.

RANDOM VARIABLE

- $\Sigma p_i = 1$.
- $E(X) = \Sigma x_i p_i$.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.