FREE NOTES · NCERT + JEE LEVEL

Probability

Conditional probability, multiplication theorem, independent events, theorem of total probability, Bayes' theorem, random variables and their probability distributions, and the mean (expectation) of a random variable.

01 Conditional probability
02 Multiplication theorem
03 Independent events
04 Theorem of total probability
05 Bayes' theorem
06 Random variables & distributions
07 Mean (expectation)
08 JEE Extra + Tricks
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01 Conditional probability

$$P(E|F) = P(E \cap F)/P(F),\ P(F) \neq 0$$

Probability of E given that F has happened. Properties: P(S|F) = P(F|F) = 1; P(E′|F) = 1 − P(E|F); P(A ∪ B|F) = P(A|F) + P(B|F) − P(A ∩ B|F).

02 Multiplication theorem

$$P(E \cap F) = P(E)\cdot P(F|E) = P(F)\cdot P(E|F)$$

$$P(E \cap F \cap G) = P(E)\cdot P(F|E)\cdot P(G|E \cap F)$$

Used for drawing without replacement: P(two aces) = 4/52 × 3/51.

03 Independent events

$$E, F \text{ independent} \Leftrightarrow P(E \cap F) = P(E)\cdot P(F)$$

Then P(E|F) = P(E). If E, F independent, so are E′ and F, E and F′, E′ and F′. P(at least one of A, B) = 1 − P(A′)P(B′).

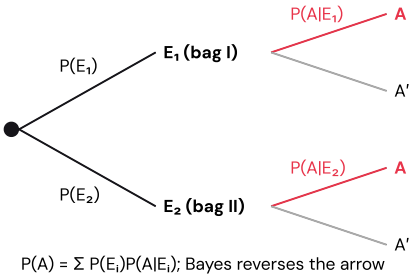
DON'T CONFUSE

Mutually exclusive: P(E ∩ F) = 0 (can't happen together). Independent: one doesn't affect the other. Two events with non-zero probabilities can't be both.

04 Theorem of total probability

If $E_1, \ldots, E_n$ partition S (mutually exclusive, exhaustive, non-zero), for any event A:

$$P(A) = \Sigma\, P(E_i)\cdot P(A|E_i)$$



Tree diagram for total probability and Bayes

05 Bayes' theorem

$$P(E_i|A) = P(E_i)P(A|E_i) \,/\, \Sigma_j\, P(E_j)P(A|E_j)$$

"Given the result A, which cause E_i produced it?" $P(E_i)$ = prior, $P(E_i|A)$ = posterior.

06 Random variables & distributions

A **random variable** X assigns a real number to each outcome (e.g. number of heads). Its **probability distribution** lists each value x_i with $P(x_i)$, where each $p_i \geq 0$ and $\Sigma p_i = 1$.

X (heads in 2 tosses)	0	1	2
P(X)	1/4	1/2	1/4

07 Mean (expectation)

$$E(X) = \mu = \Sigma\, x_i\, p_i$$

Above: E(X) = 0 + ½ + ½ = 1. (JEE extra: Var(X) = $\Sigma x_i^2 p_i - \mu^2$; binomial $P(X = r) = {}^nC_r\, p^r q^{n-r}$, mean np, variance npq.)

08 JEE Extra + Tricks

JEE LEVEL

- Bayes: $P(E_i|A) = P(E_i)P(A|E_i)/\Sigma P(E_j)P(A|E_j)$ (tree diagram helps).
- Independent ⇒ $P(A \cap B) = P(A)P(B)$; mutually exclusive ⇒ $P(A \cap B) = 0$ (cannot be both if $P(A), P(B) > 0$).
- Binomial distribution (JEE): $P(X = r) = {}^nC_r\, p^r\, q^{n-r}$; mean np, variance npq.
- Random variable: mean $\Sigma x \cdot p$, variance $\Sigma x^2 p - (\text{mean})^2$.
- Problem trick: "at least one success in n trials" = $1 - q^n$.

09 Solved examples

Q1. $P(A) = 0.8$, $P(B) = 0.5$, $P(B|A) = 0.4$. Find $P(A \cap B)$ and $P(A|B)$.

$P(A \cap B) = 0.8 \times 0.4 = 0.32$; $P(A|B) = 0.32/0.5 =$ **0.64**.

Q2. A die is thrown twice. Given the sum is 6, P(one of them is 4)?

Sum 6: (1,5),(2,4),(3,3),(4,2),(5,1) ⇒ 2 have a 4 ⇒ **2/5**.

Q3. Bag I: 3 red, 4 black; bag II: 5 red, 6 black. A bag is chosen at random and a red ball drawn. P(from bag II)?

$P = (½ \times 5/11) / (½ \times 3/7 + ½ \times 5/11) = (5/11)/(3/7 + 5/11) = 35/68$.

Q4. A and B independent, $P(A) = 0.3$, $P(B) = 0.4$. P(neither)?

$0.7 \times 0.6 =$ **0.42**.

10 Common mistakes

- Writing $P(A|B) = P(A \cap B)/P(A)$ (denominator is $P(B)$).
- Treating mutually exclusive events as independent.
- Forgetting the prior $P(E_i)$ in Bayes.
- Distribution whose probabilities don't add to 1.

11 Practice questions

1. If $P(A) = 1/2$, $P(B) = 1/3$ and A, B independent, find $P(A \cup B)$.
2. Two cards drawn without replacement. P(both kings)?
3. Find k: $P(X = 0, 1, 2) = k, 2k, 3k$.
4. Mean of the number on a fair die.

ANSWERS

1) 2/3 · 2) 1/221 · 3) 1/6 · 4) 3.5

12 Quick revision: 5 minutes before the exam

CONDITIONAL

- $P(E|F) = P(E \cap F)/P(F)$.
- $P(E \cap F) = P(E)P(F|E)$.

INDEPENDENT

- $P(E \cap F) = P(E)P(F)$.
- Not the same as exclusive.

TOTAL & BAYES

- $P(A) = \Sigma P(E_i)P(A|E_i)$.
- $P(E_i|A)$ = term/total.

RANDOM VARIABLE

- $\Sigma p_i = 1$.
- $E(X) = \Sigma x_i p_i$.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.