FREE NOTES · NCERT + JEE LEVEL

Linear Programming

What a linear programming problem is, objective function and constraints, the graphical method, feasible region, corner point method, bounded and unbounded regions, and how to formulate word problems.

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01 Key terms

- **Objective function** Z = ax + by (to maximise profit or minimise cost).
- **Decision variables** x, y (how many of each item).
- **Constraints**: linear inequalities from limits on resources.
- **Non-negative constraints**: x ≥ 0, y ≥ 0.
- **Feasible solution**: any point satisfying all constraints. **Optimal solution**: feasible point giving the best Z.

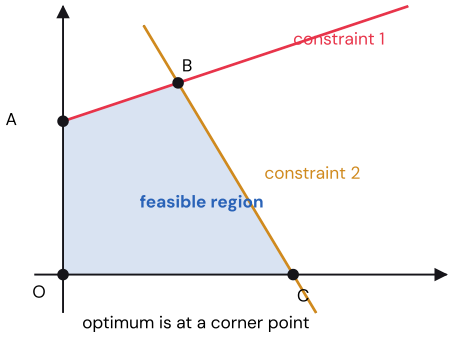
02 Formulating an LPP

1. Define x and y clearly in words.
2. Write Z (profit/cost) in x, y.
3. Translate each limit (time, material, budget) into an inequality.
4. Add x, y ≥ 0.

Example: a dealer can buy fans (₹360) and sewing machines (₹240), budget ₹5760, space for 20 items, profit ₹22 and ₹18. Max Z = 22x + 18y; 360x + 240y ≤ 5760, x + y ≤ 20, x, y ≥ 0.

03 Feasible region

Draw each constraint as a line, shade the side satisfying it (test (0, 0)). The common region is the **feasible region**. It is always a convex polygon (possibly unbounded).



Feasible region OABC and its corner points

04 Corner point method

FUNDAMENTAL THEOREM

If an optimal value exists, it occurs at a **corner point** (vertex) of the feasible region.

1. Find the feasible region and all its corner points (solve pairs of lines).
2. Evaluate Z at each corner.
3. Bounded region: largest Z = max, smallest = min.
4. If two corners give the same optimum, every point on the segment joining them is optimal.

05 Unbounded feasible regions

If the region is unbounded, the max or min may not exist. Suppose M is the largest Z at corners: draw $ax + by > M$. If this open half-plane has **no** point in common with the region, M is the maximum; otherwise no maximum. Similarly for minimum with $ax + by < m$.

06 Types of LPP

Type	Typical goal
Manufacturing	Max profit given machine hours, labour
Diet	Min cost meeting nutrient needs
Transportation	Min transport cost from factories to depots
Allocation	Best use of land, money

07 Exam presentation (full marks)

- "Let x = … and y = …" in words.
- Objective function and all constraints in a box.
- Neat graph with each line labelled and the region shaded.
- Table: corner point | value of Z.
- Final sentence answering the question (e.g. "8 fans and 12 machines give max profit ₹392").

08 JEE Extra + Tricks

JEE LEVEL

- Optimal value of LPP lies at a corner point of the feasible region (corner point theorem).
- Unbounded region: check with an extra inequality (e.g. $Z < \text{value}$) to confirm max/min exists.
- Iso-profit line method: slide line $Z = k$ parallel until it last touches the region.

09 Solved examples

Q1. Maximise $Z = 3x + 4y$ subject to $x + y \le 4$, $x, y \ge 0$.

Corners (0,0): 0; (4,0): 12; (0,4): 16 ⇒ max **16** at (0, 4).

Q2. Minimise $Z = -3x + 4y$ subject to $x + 2y \le 8$, $3x + 2y \le 12$, $x, y \ge 0$.

Corners (0,0): 0; (4,0): -12; (2,3): 6; (0,4): 16 ⇒ min **−12** at (4, 0).

Q3. Fan/sewing machine problem above.

Constraints reduce to $3x + 2y \le 48$, $x + y \le 20$. Corners (0,0): 0; (16,0): 352; (8,12): 392; (0,20): 360 ⇒ max ₹**392** with 8 fans, 12 machines.

10 Common mistakes

- Missing a corner point (check every intersection on the boundary).
- Declaring a max in an unbounded region without the half-plane check.
- Shading the wrong side (always test a point).
- Forgetting x, y ≥ 0.

11 Practice questions

1. Max Z = 5x + 3y: 3x + 5y ≤ 15, 5x + 2y ≤ 10, x, y ≥ 0.
2. Min Z = 200x + 500y: x + 2y ≥ 10, 3x + 4y ≤ 24, x, y ≥ 0.
3. Where can the optimum of an LPP lie?

ANSWERS

1) 235/19 at (20/19, 45/19) · 2) 2300 at (4, 3) · 3) At a corner point of the feasible region

12 Quick revision: 5 minutes before the exam

SETUP

- Define x, y.
- Z + constraints + x, y ≥ 0.

GRAPH

- Test (0, 0) to shade.
- Common region = feasible.

SOLVE

- Z at every corner.
- Best = optimum.

UNBOUNDED

- Check Z > M half-plane.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.