FREE NOTES · NCERT + JEE LEVEL

Three Dimensional Geometry

Line ke direction cosines aur ratios, vector aur Cartesian form me line ki equation, do lines ke beech angle, aur skew aur parallel lines ke beech shortest distance.

01 Direction cosines & direction ratios
02 Do points se guzarti line
03 Line ki equation
04 Do lines ke beech angle
05 Skew lines
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01 Direction cosines & direction ratios

- Line x, y, z axes se α, β, γ angle banaye to: **l = cos α, m = cos β, n = cos γ**, aur $l^2 + m^2 + n^2 = 1$.
- **Direction ratios** a, b, c, l, m, n ke proportional: $l = a/\sqrt{(a^2 + b^2 + c^2)}$, wagaira.
- Line ke DCs ke do sets (l, m, n) aur (−l, −m, −n).

02 Do points se guzarti line

PQ ke DRs: $x_2 - x_1, y_2 - y_1, z_2 - z_1$

DCs: $(x_2 - x_1)/PQ, (y_2 - y_1)/PQ, (z_2 - z_1)/PQ$

03 Line ki equation

Diya hai	Vector form	Cartesian form
Point a, b ke parallel	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$	$(x - x_1)/a = (y - y_1)/b = (z - z_1)/c$
Do points a, b	$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$	$(x - x_1)/(x_2 - x_1) = (y - y_1)/(y_2 - y_1) = (z - z_1)/(z_2 - z_1)$

Line pe koi bhi general point: $(x_1 + a\lambda, y_1 + b\lambda, z_1 + c\lambda)$. DRs padhne se pehle x, y, z ka coefficient 1 karo.

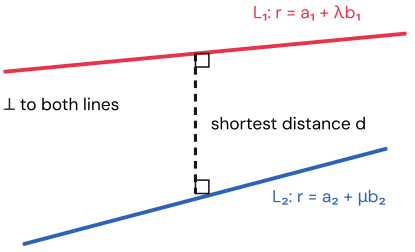
04 Do lines ke beech angle

$$\cos\theta = |a_1a_2 + b_1b_2 + c_1c_2| \,/\, (\sqrt{(a_1^2+b_1^2+c_1^2)}\,\sqrt{(a_2^2+b_2^2+c_2^2)})$$

Vector form: $\cos\theta = |\mathbf{b_1}\cdot\mathbf{b_2}|/(|\mathbf{b_1}||\mathbf{b_2}|)$. Perpendicular: $a_1a_2 + b_1b_2 + c_1c_2 = 0$. Parallel: $a_1/a_2 = b_1/b_2 = c_1/c_2$.

05 Skew lines

Space me aisi lines jo **na parallel na intersect** (alag planes me). Shortest distance common perpendicular ke along.



Skew lines ke beech shortest distance

06 Shortest distance

$$d = |(b_1 \times b_2)\cdot(a_2 - a_1)| \,/\, |b_1 \times b_2|$$

Skew lines $r = a_1 + \lambda b_1$ aur $r = a_2 + \mu b_2$ ke liye. $d = 0 \Rightarrow$ lines intersect karti.

$$d = |b \times (a_2 - a_1)| \,/\, |b|$$

Parallel lines $r = a_1 + \lambda b$ aur $r = a_2 + \mu b$ ke liye.

07 Formula sheet

DCs	$l^2 + m^2 + n^2 = 1$
Line	$r = a + \lambda b;\ (x-x_1)/a = (y-y_1)/b = (z-z_1)/c$
Angle	$\cos\theta = \lvert b_1\cdot b_2\rvert/(\lvert b_1\rvert\lvert b_2\rvert)$
SD skew	$\lvert(b_1\times b_2)\cdot(a_2-a_1)\rvert/\lvert b_1\times b_2\rvert$
SD parallel	$\lvert b\times(a_2-a_1)\rvert/\lvert b\rvert$

08 JEE Extra + Tricks

JEE LEVEL

- Skew lines $r = a_1 + \lambda b_1$, $r = a_2 + \mu b_2$ ki shortest distance: $|(a_2 - a_1)\cdot(b_1 \times b_2)|/|b_1 \times b_2|$; parallel lines: $|(a_2 - a_1) \times b|/|b|$.
- Shortest distance = 0 $\Rightarrow$ lines milti (coplanar, parallel nahi).
- JEE planes: $ax + by + cz + d = 0$; point se doori $|ax_1 + by_1 + cz_1 + d|/\sqrt{(a^2 + b^2 + c^2)}$; planes ka angle normals se.
- Do planes ke intersection se plane: $P_1 + \lambda P_2 = 0$; plane me point ki image 2D wale ratio trick se.
- Line $r = a + \lambda b$ plane $r\cdot n = d$ me $\Leftrightarrow$ $b\cdot n = 0$ aur $a\cdot n = d$.

09 Solved examples

Q1. Axes se barabar angle banane wali line ke DCs.

$3l^2 = 1 \Rightarrow l = m = n = \pm 1/\sqrt{3}$.

Q2. (5, 2, -4) se $3\hat{i} + 2\hat{j} - 8\hat{k}$ ke parallel line ki equation.

$r = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$; $(x - 5)/3 = (y - 2)/2 = (z + 4)/(-8)$.

Q3. $(x + 3)/3 = (y - 1)/5 = (z + 3)/4$ aur $(x + 1)/1 = (y - 4)/1 = (z - 5)/2$ ke beech angle.

$\cos\theta = (3 + 5 + 8)/(\sqrt{50}\,\sqrt{6}) = 16/(10\sqrt{3}) = 8\sqrt{3}/15$.

Q4. $r = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$ aur $r = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ ke beech SD.

$b_1 \times b_2 = -3\hat{i} + 0\hat{j} + 3\hat{k}$, $a_2 - a_1 = \hat{i} - 3\hat{j} - 2\hat{k} \Rightarrow |-3 - 6|/(3\sqrt{2}) = \mathbf{3/\sqrt{2}}$.

10 Common mistakes

- (2x − 1)/3 se DR 3 padh lena ((x − ½)/(3/2) likho).
- Parallel lines pe skew wala formula lagana ($b_1 \times b_2 = 0$).
- cos θ me modulus bhool jaana.
- r = a + λb me a (point) aur b (direction) mix karna.

11 Practice questions

1. l = 1/2, m = 1/2 ho to n.
2. DRs 1, 2, 3 aur −2, −4, −6 wali lines parallel?
3. (x − 1)/(−3) = (y − 2)/(2k) = (z − 3)/2 aur (x − 1)/(3k) = (y − 1)/1 = (z − 6)/(−5) perpendicular ho to k.
4. Origin aur (5, −2, 3) se line ki equation.

ANSWERS

1) ±1/√2 · 2) Haan · 3) k = −10/7 · 4) x/5 = y/(−2) = z/3

12 Quick revision: exam se 5 minute pehle

DIRECTION

- $l^2 + m^2 + n^2 = 1$.
- DRs ∝ DCs.

LINE

- r = a + λb.
- x, y, z ka coefficient 1.

ANGLE

- ⊥: $\Sigma a_i a_j = 0$.
- ∥: ratios barabar.

DOORI

- Skew: $|(b_1 \times b_2) \cdot (a_2 - a_1)| / |b_1 \times b_2|$.
- d = 0 ⇒ intersect.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.