FREE NOTES · NCERT + JEE LEVEL

Vector Algebra

Scalars aur vectors, vectors ke types, direction cosines, addition aur scalar multiplication, components, section formula, dot product, projection, cross product aur areas.

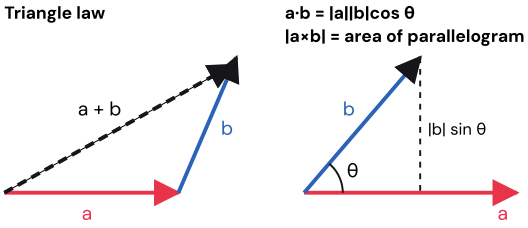
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01 Scalars, vectors & types

Vector me magnitude aur direction dono (displacement, force); **scalar** me sirf magnitude. P(x, y, z) ka position vector: $r = x\hat{i} + y\hat{j} + z\hat{k}$, $|r| = \sqrt{(x^2 + y^2 + z^2)}$.

Type	Matlab
Zero vector	Magnitude 0, koi bhi direction
Unit vector â	a/|a|, magnitude 1
Co-initial	Same starting point
Collinear	Ek hi line ke parallel ($a = \lambda b$)
Equal	Same magnitude aur direction
Negative	Same magnitude, ulti direction

02 Addition & scalar multiplication



Triangle law, aur dot-cross product me use hone wala angle

- **Triangle law:** AB + BC = AC. **Parallelogram law** bhi wahi deta.
- Commutative aur associative. a + (−a) = 0.
- λa ka magnitude |λ||a|; λ > 0 ho to same direction, λ < 0 ho to ulti.

03 Components & direction cosines

$\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k};\ \ AB = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

Direction cosines $l = a_1/|a|,\ m = a_2/|a|,\ n = a_3/|a|;\ l^2 + m^2 + n^2 = 1$

Direction ratios = a_1, a_2, a_3 (koi bhi multiple).

04 Section formula (vectors)

Andar $m : n \Rightarrow r = (mb + na)/(m + n)$; bahar $\Rightarrow (mb - na)/(m - n)$

Midpoint $= (a + b)/2$

05 Dot (scalar) product

$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$$

- $\mathbf{a}\cdot\mathbf{b} = 0 \Leftrightarrow \mathbf{a} \perp \mathbf{b}$ (non-zero vectors).
- $\hat{i}\cdot\hat{i} = 1, \hat{i}\cdot\hat{j} = 0$. $\mathbf{a}\cdot\mathbf{a} = |\mathbf{a}|^2$.
- $\cos\theta = \mathbf{a}\cdot\mathbf{b}/(|\mathbf{a}||\mathbf{b}|)$.
- a ka b pe **projection** = $\mathbf{a}\cdot\mathbf{b}/|\mathbf{b}|$.
- Commutative aur distributive.

06 Cross (vector) product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{n}$$

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

- $\hat{n}$, a aur b dono ke $\perp$ (right-hand rule).
- $\mathbf{a}\times\mathbf{b} = -(\mathbf{b}\times\mathbf{a})$ (commutative nahi). $\mathbf{a}\times\mathbf{a} = 0$. $\mathbf{a}\times\mathbf{b} = 0 \Leftrightarrow \mathbf{a} \parallel \mathbf{b}$.
- $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$.
- Parallelogram ka area = $|\mathbf{a}\times\mathbf{b}|$; triangle = $\frac{1}{2}|\mathbf{a}\times\mathbf{b}|$; diagonals d_1, d_2 se: $\frac{1}{2}|\mathbf{d}_1\times\mathbf{d}_2|$.

07 Formula sheet

Unit vector	$\mathbf{a}/\lvert\mathbf{a}\rvert$
Dot	$\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta = \Sigma a_i b_i$
a ka b pe projection	$\mathbf{a}\cdot\mathbf{b}/\lvert\mathbf{b}\rvert$
Cross magnitude	$\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$
Triangle ka area	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$
DCs	$l^2 + m^2 + n^2 = 1$

08 JEE Extra + Tricks

JEE LEVEL

- Scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$ = parallelepiped ka volume; zero ⇒ coplanar.
- Vector triple product $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$.
- $\vec{a}$ ka $\vec{b}$ pe projection = $(\vec{a}\cdot\vec{b})/|\vec{b}|$; triangle area $\frac{1}{2}|\vec{a}\times\vec{b}|$; parallelogram $|\vec{a}\times\vec{b}|$ (diagonals se $\frac{1}{2}|\vec{d}_1\times\vec{d}_2|$).
- $|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$; $\vec{a}\perp\vec{b} \Leftrightarrow |\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$.
- $\vec{a}$ aur $\vec{b}$ ke bisector ki unit vector: $(\hat{a}+\hat{b})/|\hat{a}+\hat{b}|$.

09 Solved examples

Q1. a = 2î + 3ĵ + k̂ ki disha me unit vector.

|a| = √14 ⇒ (2î + 3ĵ + k̂)/√14.

Q2. î − 2ĵ + 3k̂ aur 3î − 2ĵ + k̂ ke beech angle.

a·b = 3 + 4 + 3 = 10; |a| = |b| = √14 ⇒ cos θ = 10/14 = 5/7 ⇒ θ = $\cos^{-1}(5/7)$.

Q3. a = 3î + ĵ + 4k̂ aur b = î − ĵ + k̂ ho to |a × b|.

a × b = î(1 + 4) − ĵ(3 − 4) + k̂(−3 − 1) = 5î + ĵ − 4k̂ ⇒ **√42**.

Q4. Vertices A(1, 1, 2), B(2, 3, 5), C(1, 5, 5) wale triangle ka area.

AB = î + 2ĵ + 3k̂, AC = 4ĵ + 3k̂; AB × AC = −6î − 3ĵ + 4k̂ ⇒ area = ½√61.

10 Common mistakes

- a × b = b × a likhna.
- a ke b pe projection me denominator |a| lena (|b| hai).
- a·b ko vector ya a × b ko scalar kehna.
- Cross product determinant me beech wale (ĵ) term ka sign galat.

11 Practice questions

1. 2î + λĵ + k̂, î − 2ĵ + 3k̂ ke ⊥ ho to λ.
2. î + 3ĵ + 7k̂ ka 7î − ĵ + 8k̂ pe projection.
3. |a| = 2, |b| = 3, a·b = 3 ho to θ.
4. î + ĵ aur ĵ + k̂ dono ke ⊥ unit vector.

ANSWERS

1) 5/2 · 2) 60/√114 · 3) π/3 · 4) (î − ĵ + k̂)/√3

12 Quick revision: exam se 5 minute pehle

BASICS

- $\hat{a} = a/|a|$.
- $l^2 + m^2 + n^2 = 1$.

DOT

- $= 0 \Rightarrow$ perpendicular.
- Projection $a \cdot b/|b|$.

CROSS

- $= 0 \Rightarrow$ parallel.
- $a \times b = -b \times a$.

AREAS

- Parallelogram $|a \times b|$.
- Triangle $\frac{1}{2}|a \times b|$.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.