FREE NOTES · NCERT + JEE LEVEL

Vector Algebra

Scalars and vectors, types of vectors, direction cosines, addition and scalar multiplication, components, section formula, dot product, projection, cross product and areas.

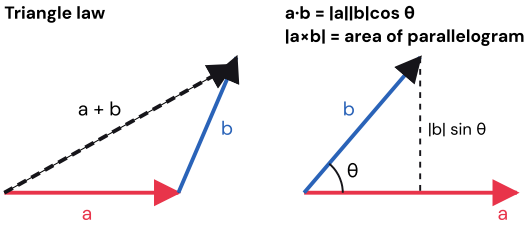
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01 Scalars, vectors & types

A **vector** has magnitude and direction (displacement, force); a **scalar** has only magnitude. Position vector of P(x, y, z): $r = x\hat{i} + y\hat{j} + z\hat{k}$, $|r| = \sqrt{(x^2 + y^2 + z^2)}$.

Type	Meaning
Zero vector	Magnitude 0, any direction
Unit vector â	a/|a|, magnitude 1
Co-initial	Same starting point
Collinear	Parallel to the same line (a = λb)
Equal	Same magnitude and direction
Negative	Same magnitude, opposite direction

02 Addition & scalar multiplication



Triangle law, and the angle used in dot and cross products

- **Triangle law:** AB + BC = AC. **Parallelogram law** gives the same result.
- Commutative and associative. a + (−a) = 0.
- λa has magnitude |λ||a|; same direction if λ > 0, opposite if λ < 0.

03 Components & direction cosines

$\mathbf{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k};\ \ AB = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

Direction cosines $l = a_1/|a|,\ m = a_2/|a|,\ n = a_3/|a|;\ l^2 + m^2 + n^2 = 1$

Direction ratios = a_1, a_2, a_3 (any multiple).

04 Section formula (vectors)

Internal $m : n \Rightarrow r = (mb + na)/(m + n)$; external $\Rightarrow (mb - na)/(m - n)$

Midpoint $= (a + b)/2$

05 Dot (scalar) product

$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$$

- $\mathbf{a}\cdot\mathbf{b} = 0 \Leftrightarrow \mathbf{a} \perp \mathbf{b}$ (non-zero vectors).
- $\hat{i}\cdot\hat{i} = 1, \hat{i}\cdot\hat{j} = 0$. $\mathbf{a}\cdot\mathbf{a} = |\mathbf{a}|^2$.
- $\cos\theta = \mathbf{a}\cdot\mathbf{b}/(|\mathbf{a}||\mathbf{b}|)$.
- **Projection** of a on b = $\mathbf{a}\cdot\mathbf{b}/|\mathbf{b}|$.
- Commutative and distributive.

06 Cross (vector) product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{n}$$

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

- $\hat{n} \perp$ to both a and b (right-hand rule).
- $\mathbf{a}\times\mathbf{b} = -(\mathbf{b}\times\mathbf{a})$ (not commutative). $\mathbf{a}\times\mathbf{a} = 0$. $\mathbf{a}\times\mathbf{b} = 0 \Leftrightarrow \mathbf{a} \parallel \mathbf{b}$.
- $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$.
- Area of parallelogram = $|\mathbf{a}\times\mathbf{b}|$; triangle = $\frac{1}{2}|\mathbf{a}\times\mathbf{b}|$; with diagonals d_1, d_2: $\frac{1}{2}|\mathbf{d}_1\times\mathbf{d}_2|$.

07 Formula sheet

Unit vector	$\mathbf{a}/\lvert\mathbf{a}\rvert$
Dot	$\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta = \Sigma a_i b_i$
Projection of a on b	$\mathbf{a}\cdot\mathbf{b}/\lvert\mathbf{b}\rvert$
Cross magnitude	$\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$
Area of triangle	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$
DCs	$l^2 + m^2 + n^2 = 1$

08 JEE Extra + Tricks

JEE LEVEL

- Scalar triple product $[a\ b\ c] = a\cdot(b \times c)$ = volume of parallelepiped; zero $\Rightarrow$ coplanar.
- Vector triple product $a \times (b \times c) = (a\cdot c)b - (a\cdot b)c$.
- Projection of a on b = $(a\cdot b)/|b|$; area of triangle $\tfrac{1}{2}|a \times b|$; parallelogram $|a \times b|$ (or $\tfrac{1}{2}|d_1 \times d_2|$ with diagonals).
- $|a + b|^2 + |a - b|^2 = 2(|a|^2 + |b|^2)$; $a \perp b \Leftrightarrow |a + b| = |a - b|$.
- Unit vector along bisector of a and b: $(\hat{a} + \hat{b})/|\hat{a} + \hat{b}|$.

09 Solved examples

Q1. Unit vector along $a = 2\hat{i} + 3\hat{j} + \hat{k}$.

$|a| = \sqrt{14} \Rightarrow (2\hat{i} + 3\hat{j} + \hat{k})/\sqrt{14}$.

Q2. Angle between $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

$a \cdot b = 3 + 4 + 3 = 10$; $|a| = |b| = \sqrt{14} \Rightarrow \cos\theta = 10/14 = 5/7 \Rightarrow \theta = \cos^{-1}(5/7)$.

Q3. Find $|a \times b|$ if $a = 3\hat{i} + \hat{j} + 4\hat{k}$ and $b = \hat{i} - \hat{j} + \hat{k}$.

$a \times b = \hat{i}(1 + 4) - \hat{j}(3 - 4) + \hat{k}(-3 - 1) = 5\hat{i} + \hat{j} - 4\hat{k} \Rightarrow$ **$\sqrt{42}$**.

Q4. Area of triangle with vertices A(1, 1, 2), B(2, 3, 5), C(1, 5, 5).

$AB = \hat{i} + 2\hat{j} + 3\hat{k}$, $AC = 4\hat{j} + 3\hat{k}$; $AB \times AC = -6\hat{i} - 3\hat{j} + 4\hat{k} \Rightarrow$ area $= \frac{1}{2}\sqrt{61}$.

10 Common mistakes

- Writing $a \times b = b \times a$.
- Projection of a on b with $|a|$ in the denominator (it's $|b|$).
- Calling $a \cdot b$ a vector or $a \times b$ a scalar.
- Wrong sign of the middle ($\hat{j}$) term in the cross product determinant.

11 Practice questions

1. Find λ if $2\hat{i} + \lambda\hat{j} + \hat{k} \perp \hat{i} - 2\hat{j} + 3\hat{k}$.
2. Projection of $\hat{i} + 3\hat{j} + 7\hat{k}$ on $7\hat{i} - \hat{j} + 8\hat{k}$.
3. If $|a| = 2$, $|b| = 3$, $a \cdot b = 3$, find θ.
4. Unit vector $\perp$ to both $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$.

ANSWERS

1) 5/2 · 2) 60/√114 · 3) π/3 · 4) $(\hat{i} - \hat{j} + \hat{k})/\sqrt{3}$

12 Quick revision: 5 minutes before the exam

BASICS

- $\hat{a} = a/|a|$.
- $l^2 + m^2 + n^2 = 1$.

DOT

- $= 0 \Rightarrow$ perpendicular.
- Projection $a \cdot b/|b|$.

CROSS

- $= 0 \Rightarrow$ parallel.
- $a \times b = -b \times a$.

AREAS

- Parallelogram $|a \times b|$.
- Triangle $\frac{1}{2}|a \times b|$.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.