FREE NOTES · NCERT + JEE LEVEL

Differential Equations

Order aur degree, general aur particular solutions, differential equation banana, aur variable-separable, homogeneous aur integrating factor wali linear first-order equations solve karna.

01 Order & degree
02 General & particular solutions
03 Differential equation banana
04 Variable separable
05 Homogeneous equations
06 Linear first-order equations
07 Kaun sa method?
08 JEE Extra + Tricks
09 Solved examples
10 Common mistakes
11 Practice questions
12 Quick revision: exam se 5 minute pehle

01 Order & degree

- **Order** = sabse oonche derivative ka order.
- **Degree** = equation ko derivatives me polynomial banane ke baad sabse oonche derivative ki power. Agar derivative sin, log, e ke andar ho to defined nahi.

Equation	Order	Degree
$dy/dx = \cos x$	1	1
$(d^2y/dx^2)^3 + (dy/dx)^2 + y = 0$	2	3
$y'' + \sin(y') = 0$	2	Defined nahi

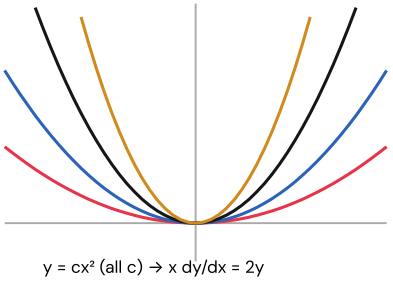
02 General & particular solutions

General solution me order jitne arbitrary constants. **Particular solution**: di gayi conditions se constants fix (initial value problem).

03 Differential equation banana

n constants wali curves ki family di ho: n baar differentiate karo aur constants hatao. Bani DE ka order n.

Example: $y = cx^2 \Rightarrow y' = 2cx \Rightarrow c = y/x^2 \Rightarrow \mathbf{xy' = 2y}$.



Ek differential equation poori family batati

04 Variable separable

$$dy/dx = g(x)h(y) \Rightarrow \int dy/h(y) = \int g(x)\,dx + C$$

Example: $dy/dx = (1 + y^2)/(1 + x^2) \Rightarrow \tan^{-1}y = \tan^{-1}x + C$.

05 Homogeneous equations

$dy/dx = F(x, y)$ jahan $F(\lambda x, \lambda y) = F(x, y)$ (y/x ka function). **$y = vx$ rakho** $\Rightarrow dy/dx = v + x\,dv/dx$, phir variables alag karo. (dx/dy form ho to $x = vy$.)

Check: $(x^2 + y^2)/(xy)$ degree 0 ka homogeneous hai.

06 Linear first-order equations

$dy/dx + P\,y = Q$ (P, Q x ke functions)

$$IF = e^{\int P\,dx};\ \ y \cdot IF = \int Q \cdot IF\,dx + C$$

Form $dx/dy + P_1 x = Q_1$ ho (P_1, Q_1 y ke functions): IF = e^{∫P_1 dy}, x·IF = ∫Q_1·IF dy + C.

07 Kaun sa method?

Kya dekho	Method
f(x)dx = g(y)dy me alag ho sake	Variable separable
Saare terms same total degree (y/x form)	Homogeneous, y = vx
y aur y′ sirf power 1 me	Linear, integrating factor

08 JEE Extra + Tricks

JEE LEVEL

- Order = sabse badi derivative; degree = derivatives me polynomial banane ke baad uski power (sin, e ke andar y′ ho to undefined).
- Arbitrary constants ki sankhya = DE ka order.
- Linear DE $dy/dx + Py = Q$: $IF = e^{\int P\,dx}$; $y \cdot IF = \int Q \cdot IF\,dx + C$.
- Bernoulli equation $dy/dx + Py = Qy^n$: $z = y^{1-n}$ rakhkar linear (JEE).
- Exact-differential tricks: $d(xy) = x\,dy + y\,dx$; $d(y/x) = (x\,dy - y\,dx)/x^2$; $d(\ln(xy)) = (x\,dy + y\,dx)/(xy)$.

09 Solved examples

Q1. $dy/dx = e^{x+y}$ solve karo.

$e^{-y}\,dy = e^x\,dx \Rightarrow -e^{-y} = e^x + C \Rightarrow \mathbf{e^x + e^{-y} = C}$.

Q2. $(x - y)dy - (x + y)dx = 0$ solve karo.

$dy/dx = (x + y)/(x - y)$; $y = vx \Rightarrow x\,dv/dx = (1 + v^2)/(1 - v) \Rightarrow \tan^{-1}(y/x) = \tfrac{1}{2}\log(x^2 + y^2) + C$.

Q3. $dy/dx + y/x = x^2$ solve karo.

$IF = x \Rightarrow xy = \int x^3\,dx = x^4/4 + C \Rightarrow \mathbf{y = x^3/4 + C/x}$.

Q4. $dy/dx = y \tan x$ ka particular solution, $x = 0$ pe $y = 1$.

$\log y = \log \sec x + C \Rightarrow y = C \sec x$; $y(0) = 1 \Rightarrow C = 1 \Rightarrow \mathbf{y = \sec x}$.

10 Common mistakes

- Derivative sin ya log ke andar ho phir bhi degree batana.
- Constant C bhool jaana (ya dono taraf lagana).
- Equation ko pehle dy/dx + Py = Q me likhe bina IF lagana (dy/dx ka coefficient 1 hona chahiye).
- e^{log x} ko log x likhna (ye x hai).

11 Practice questions

1. y''' + 2y'' + y' = 0 ka order aur degree.
2. 3rd-order DE ke general solution me kitne arbitrary constants?
3. dy/dx = (1 + y)/(2 + x) solve karo.
4. dy/dx − y = cos x ka IF.

ANSWERS

1) 3, 1 · 2) 3 · 3) 1 + y = C(2 + x) · 4) e^{-x}

12 Quick revision: exam se 5 minute pehle

BASICS

- Order = sabse ooncha derivative.
- Degree ke liye polynomial form.
- Constants = order.

SEPARABLE

- ∫dy/h(y) = ∫g(x)dx.

HOMOGENEOUS

- y = vx.

LINEAR

- IF = e^{∫P dx}.
- y·IF = ∫Q·IF dx + C.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.