FREE NOTES · NCERT + JEE LEVEL

Differential Equations

Order and degree, general and particular solutions, forming a differential equation, and solving variable-separable, homogeneous and linear first-order equations with the integrating factor.

01 Order & degree
02 General & particular solutions
03 Forming a differential equation
04 Variable separable
05 Homogeneous equations
06 Linear first-order equations
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01 Order & degree

- **Order** = order of the highest derivative present.
- **Degree** = power of the highest-order derivative after the equation is made polynomial in derivatives. Not defined if a derivative appears inside sin, log, e, etc.

Equation	Order	Degree
$dy/dx = \cos x$	1	1
$(d^2y/dx^2)^3 + (dy/dx)^2 + y = 0$	2	3
$y'' + \sin(y') = 0$	2	Not defined

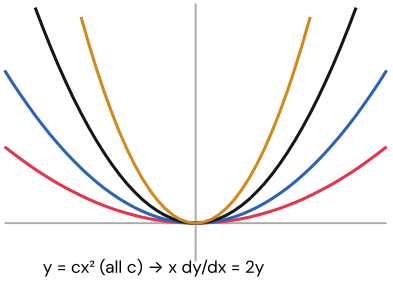
02 General & particular solutions

General solution contains as many arbitrary constants as the order. **Particular solution**: constants fixed using given conditions (initial value problem).

03 Forming a differential equation

Given a family of curves with n constants: differentiate n times and eliminate the constants. The resulting DE has order n.

Example: $y = cx^2 \Rightarrow y' = 2cx \Rightarrow c = y/x^2 \Rightarrow \mathbf{xy' = 2y}$.



$y = cx^2$ (all c) → $x\,dy/dx = 2y$

One differential equation describes the whole family

04 Variable separable

$$dy/dx = g(x)h(y) \Rightarrow \int dy/h(y) = \int g(x)\,dx + C$$

Example: $dy/dx = (1 + y^2)/(1 + x^2) \Rightarrow \tan^{-1}y = \tan^{-1}x + C$.

05 Homogeneous equations

$dy/dx = F(x, y)$ where $F(\lambda x, \lambda y) = F(x, y)$ (function of y/x). **Put $y = vx$** $\Rightarrow dy/dx = v + x\,dv/dx$, then separate variables. (If dx/dy form, put $x = vy$.)

Check: $(x^2 + y^2)/(xy)$ is homogeneous of degree 0.

06 Linear first-order equations

$dy/dx + P\,y = Q$ (P, Q functions of x)

$$\text{IF} = e^{\int P\,dx};\ \ y\cdot\text{IF} = \int Q\cdot\text{IF}\,dx + C$$

If the form is $dx/dy + P_1x = Q_1$ (P_1, Q_1 functions of y): $IF = e^{\int P_1\,dy}$, $x\cdot IF = \int Q_1\cdot IF\,dy + C$.

07 Which method to use?

Look for	Method
Can split into f(x)dx = g(y)dy	Variable separable
All terms same total degree (y/x form)	Homogeneous, y = vx
y and y′ appear only to power 1	Linear, integrating factor

08 JEE Extra + Tricks

JEE LEVEL

- Order = highest derivative; degree = power of highest derivative after making the equation polynomial in derivatives (undefined if y′ inside sin, e etc.).
- Number of arbitrary constants = order of DE formed.
- Linear DE $dy/dx + Py = Q$: $IF = e^{\int P\,dx}$; $y \cdot IF = \int Q \cdot IF\,dx + C$.
- Bernoulli's equation $dy/dx + Py = Qy^n$: put $z = y^{1-n}$ to make it linear (JEE).
- Exact-differential tricks: $d(xy) = x\,dy + y\,dx$; $d(y/x) = (x\,dy - y\,dx)/x^2$; $d(\ln(xy)) = (x\,dy + y\,dx)/(xy)$.

09 Solved examples

Q1. Solve $dy/dx = e^{x+y}$.

$e^{-y}\,dy = e^x\,dx \Rightarrow -e^{-y} = e^x + C \Rightarrow \mathbf{e^x + e^{-y} = C}$.

Q2. Solve $(x - y)dy - (x + y)dx = 0$.

$dy/dx = (x + y)/(x - y)$; $y = vx \Rightarrow x\,dv/dx = (1 + v^2)/(1 - v) \Rightarrow \tan^{-1}(y/x) = \tfrac{1}{2}\log(x^2 + y^2) + C$.

Q3. Solve $dy/dx + y/x = x^2$.

$IF = x \Rightarrow xy = \int x^3\,dx = x^4/4 + C \Rightarrow \mathbf{y = x^3/4 + C/x}$.

Q4. Find the particular solution of $dy/dx = y \tan x$, $y = 1$ when $x = 0$.

$\log y = \log \sec x + C \Rightarrow y = C \sec x$; $y(0) = 1 \Rightarrow C = 1 \Rightarrow \mathbf{y = \sec x}$.

10 Common mistakes

- Giving a degree when a derivative is inside sin or log.
- Forgetting the constant C (or adding it on both sides).
- Using IF without first writing the equation as dy/dx + Py = Q (coefficient of dy/dx must be 1).
- Writing e^{log x} as log x (it is x).

11 Practice questions

1. Order and degree of $y''' + 2y'' + y' = 0$.
2. Number of arbitrary constants in the general solution of a 3rd-order DE.
3. Solve $dy/dx = (1 + y)/(2 + x)$.
4. IF of $dy/dx - y = \cos x$.

ANSWERS

1) 3, 1 · 2) 3 · 3) $1 + y = C(2 + x)$ · 4) e^{-x}

12 Quick revision: 5 minutes before the exam

BASICS

- Order = highest derivative.
- Degree needs polynomial form.
- Constants = order.

SEPARABLE

- $\int dy/h(y) = \int g(x)dx$.

HOMOGENEOUS

- $y = vx$.

LINEAR

- IF = e^{∫P dx}.
- y·IF = ∫Q·IF dx + C.

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