FREE NOTES · NCERT + JEE LEVEL

Application of Integrals

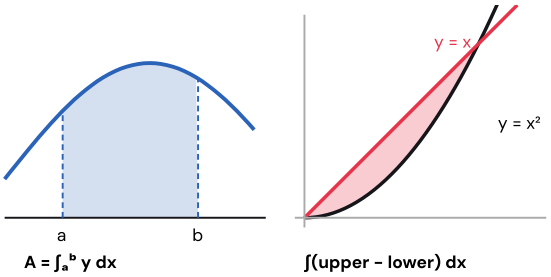
Simple curves ke neeche area, curve aur line se ghira area, circle aur ellipse ka area, aur aise sawaal step by step kaise set aur solve karein.

01 Curve ke neeche area (vertical strips)
02 Horizontal strips
03 Circle & ellipse ka area
04 Curve aur line se ghira region
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01 Curve ke neeche area (vertical strips)

$$A = \int_a^b y\,dx = \int_a^b f(x)\,dx$$

y = f(x), x-axis aur x = a, x = b ke beech area. Curve x-axis ke neeche ho to integral negative: |...| lo. Cross kare to crossing point pe todo.



Curve ke neeche area, aur do curves ke beech area

02 Horizontal strips

x = g(y), y-axis aur y = c, y = d ke beech area: $A = \int_c^d x\,dy$. Jab curve x ko y me likhna aasan ho tab use karo (jaise y-axis ke paas wale region ke liye $y^2 = 4ax$).

03 Circle & ellipse ka area

Circle $x^2 + y^2 = a^2$: $A = 4\int_0^a \sqrt{(a^2 - x^2)}\,dx = 4 \times (\pi a^2/4) = \boldsymbol{\pi a^2}$

Ellipse $x^2/a^2 + y^2/b^2 = 1$: $A = 4(b/a)\int_0^a \sqrt{(a^2 - x^2)}\,dx = \boldsymbol{\pi ab}$

$\int\sqrt{(a^2 - x^2)}\,dx = (x/2)\sqrt{(a^2 - x^2)} + (a^2/2)\sin^{-1}(x/a)$ use hota. Symmetry: ek quadrant nikalo, 4 se guna.

04 Curve aur line se ghira region

1. Dono equations saath solve karke intersection points nikalo.
2. Us interval me kaun sa graph upar hai dekho.
3. A = intersection x-values ke beech ∫ (upar - neeche) dx.

05 Do curves ke beech area

$$A = \int_a^b [f(x) - g(x)]\,dx, [a, b] \text{ pe } f \geq g$$

Curves jagah badlein to har crossing pe interval todo. Example: y = x aur y = x², 0 aur 1 pe milte; area = $\int_0^1 (x - x^2)\,dx$ = 1/6.

06 Step-by-step method

HAR AREA SAWAAL KE LIYE

1. **Rough sketch banao** aur region shade karo (iske marks milte).
2. Intersections se limits nikalo.
3. Vertical (dx) ya horizontal (dy) strips chuno.
4. Integral likho, ho sake to symmetry use karo.
5. Solve karo; "sq units" likho.

07 Standard results

Region	Area
Radius a ka circle	πa²
Ellipse x²/a² + y²/b² = 1	πab
y² = 4ax aur latus rectum x = a	8a²/3
y² = 4ax aur x² = 4ay	16a²/3
y = x² aur y = x	1/6

08 JEE Extra + Tricks

JEE LEVEL

- Curves ke beech area = intersection points ke beech $\int|f(x) - g(x)|dx$; pehle sketch.
- Ellipse ka area πab; $y^2 = 4ax$ aur $x^2 = 4ay$ ke beech = $16a^2/3$.
- Parabola $y^2 = 4ax$ aur latus rectum ke beech = $8a^2/3$.
- $x = f(y)$ aasaan ho to horizontal strips (dy).

09 Solved examples

Q1. $y = x^2$, $x = 1$, $x = 2$ aur x-axis se ghira area.

$\int_1^2 x^2\,dx = (8 - 1)/3 =$ **7/3** sq units.

Q2. Ellipse $x^2/16 + y^2/9 = 1$ ka area.

$\pi ab = \pi \times 4 \times 3 =$ **12π** sq units.

Q3. Pehle quadrant me $y^2 = 9x$, $x = 2$, $x = 4$ aur x-axis se ghira area.

$\int_2^4 3\sqrt{x}\,dx = 2[x^{3/2}]_2^4 = 2(8 - 2\sqrt{2}) =$ **16 − 4√2** sq units.

Q4. Pehle quadrant me $y = x$, x-axis aur $x^2 + y^2 = 32$ se ghira area.

(4, 4) pe milte. $A = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{(32 - x^2)}\,dx = 8 + (4\pi - 8) =$ **4π** sq units.

10 Common mistakes

- Figure na banana (marks katte aur limits galat).
- x-axis ke neeche wale area ko negative chhod dena.
- Neeche me se upar ghatana (negative aata).
- Symmetry factor se guna bhool jaana.

11 Practice questions

1. Circle $x^2 + y^2 = 4$ ka area.
2. 0 se π tak $y = \sin x$ ke neeche area.
3. $y = x^2$ aur $y = 4$ ke beech area.
4. $y = |x|$ aur $y = 1$ se ghira area.

ANSWERS

1) 4π · 2) 2 · 3) 32/3 · 4) 1

12 Quick revision: exam se 5 minute pehle

FORMULAS

- A = ∫ y dx ya ∫ x dy.
- Beech me: ∫(upar − neeche).

SHAPES

- Circle πa², ellipse πab.
- y = x² & y = x: 1/6.

METHOD

- Sketch → limits → strip → integrate.

DHYAN

- Area hamesha positive.
- Crossings pe todo.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.