FREE NOTES · NCERT + JEE LEVEL

Application of Integrals

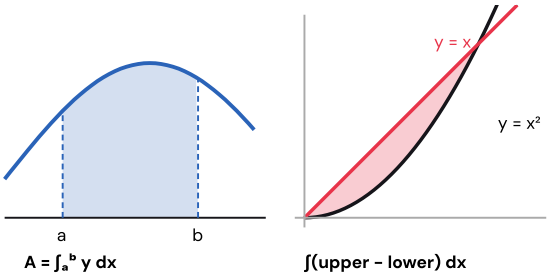
Area under simple curves, area bounded by a curve and a line, areas of circle and ellipse, and how to set up and evaluate these problems step by step.

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01 Area under a curve (vertical strips)

$$A = \int_a^b y\,dx = \int_a^b f(x)\,dx$$

Area between y = f(x), the x-axis and x = a, x = b. If the curve is below the x-axis, the integral is negative: take |...|. If it crosses, split at the crossing point.



Area under a curve, and area between two curves

02 Horizontal strips

Area between x = g(y), the y-axis and y = c, y = d: $A = \int_c^d x\,dy$. Use this when the curve is easier as x in terms of y (like $y^2 = 4ax$ for the region next to the y-axis).

03 Area of circle & ellipse

Circle $x^2 + y^2 = a^2$: $A = 4\int_0^a \sqrt{(a^2 - x^2)}\,dx = 4 \times (\pi a^2/4) = \boldsymbol{\pi a^2}$

Ellipse $x^2/a^2 + y^2/b^2 = 1$: $A = 4(b/a)\int_0^a \sqrt{(a^2 - x^2)}\,dx = \boldsymbol{\pi ab}$

Uses $\int\sqrt{(a^2 - x^2)}\,dx = (x/2)\sqrt{(a^2 - x^2)} + (a^2/2)\sin^{-1}(x/a)$. Symmetry: find one quadrant, multiply by 4.

04 Region bounded by a curve and a line

1. Find intersection points by solving the two equations together.
2. Decide which graph is on top in that interval.
3. A = ∫ (upper − lower) dx between the intersection x-values.

05 Area between two curves

$$A = \int_a^b [f(x) - g(x)]\,dx,\ f \geq g \text{ on } [a, b]$$

If the curves swap position, split the interval at each crossing. Example: $y = x$ and $y = x^2$ meet at 0 and 1; area $= \int_0^1 (x - x^2)\,dx = 1/6$.

06 Step-by-step method

FOR EVERY AREA QUESTION

1. **Draw a rough sketch** and shade the region (marks are given for it).
2. Find limits from intersections.
3. Choose vertical (dx) or horizontal (dy) strips.
4. Write the integral, use symmetry if possible.
5. Evaluate; write "sq units".

07 Standard results

Region	Area
Circle radius a	πa^2
Ellipse $x^2/a^2 + y^2/b^2 = 1$	πab
$y^2 = 4ax$ and its latus rectum $x = a$	$8a^2/3$
$y^2 = 4ax$ and $x^2 = 4ay$	$16a^2/3$
$y = x^2$ and $y = x$	1/6

08 JEE Extra + Tricks

JEE LEVEL

- Area between curves = $\int|f(x) - g(x)|dx$ between intersection points; always sketch first.
- Area of ellipse πab; area between $y^2 = 4ax$ and $x^2 = 4ay = 16a^2/3$.
- Area bounded by parabola $y^2 = 4ax$ and its latus rectum = $8a^2/3$.
- Use horizontal strips (dy) when curves are easier as $x = f(y)$.

09 Solved examples

Q1. Area bounded by $y = x^2$, $x = 1$, $x = 2$ and the x-axis.

$\int_1^2 x^2\,dx = (8 - 1)/3 =$ **7/3** sq units.

Q2. Area of the ellipse $x^2/16 + y^2/9 = 1$.

$\pi ab = \pi \times 4 \times 3 =$ **12π** sq units.

Q3. Area bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

$\int_2^4 3\sqrt{x}\,dx = 2[x^{3/2}]_2^4 = 2(8 - 2\sqrt{2}) =$ **16 − 4√2** sq units.

Q4. Area of the region in the first quadrant bounded by $y = x$, x-axis and $x^2 + y^2 = 32$.

Meet at (4, 4). $A = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{(32 - x^2)}\,dx = 8 + (4\pi - 8) =$ **4π** sq units.

10 Common mistakes

- Not drawing the figure (lose marks and get wrong limits).
- Leaving a negative answer for area below the x-axis.
- Subtracting upper from lower (gets negative).
- Forgetting to multiply by symmetry factor.

11 Practice questions

1. Area enclosed by the circle $x^2 + y^2 = 4$.
2. Area under $y = \sin x$ from 0 to π.
3. Area between $y = x^2$ and $y = 4$.
4. Area bounded by $y = |x|$ and $y = 1$.

ANSWERS

1) 4π · 2) 2 · 3) 32/3 · 4) 1

12 Quick revision: 5 minutes before the exam

FORMULAS

- $A = \int y\,dx$ or $\int x\,dy$.
- Between: $\int$(upper − lower).

SHAPES

- Circle πa^2, ellipse πab.
- $y = x^2$ & $y = x$: 1/6.

METHOD

- Sketch → limits → strip → integrate.

CAREFUL

- Area always positive.
- Split at crossings.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.