FREE NOTES · NCERT + JEE LEVEL

Integrals

Integration as anti-derivative, standard formulas, substitution, trigonometric identities, partial fractions, integration by parts, special integrals, definite integrals, the fundamental theorem and properties of definite integrals.

01 Integration as anti-derivative
02 Standard formulas
03 Integration by substitution
04 Partial fractions
05 Integration by parts
06 Special integrals
07 Definite integrals & properties
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09 Solved examples
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01 Integration as anti-derivative

If d/dx F(x) = f(x), then ∫f(x) dx = F(x) + C (indefinite integral; C = constant of integration). ∫[f ± g] = ∫f ± ∫g; ∫kf = k∫f.

02 Standard formulas

$\int f(x)\,dx$	Result	$\int f(x)\,dx$	Result
x^n $(n \neq -1)$	$x^{n+1}/(n+1)$	$1/x$	$\log\|x\|$
e^x	e^x	a^x	$a^x/\log a$
$\sin x$	$-\cos x$	$\cos x$	$\sin x$
$\sec^2 x$	$\tan x$	$\operatorname{cosec}^2 x$	$-\cot x$
$\sec x \tan x$	$\sec x$	$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x$
$\tan x$	$\log\|\sec x\|$	$\cot x$	$\log\|\sin x\|$
$\sec x$	$\log\|\sec x + \tan x\|$	$\operatorname{cosec} x$	$\log\|\operatorname{cosec} x - \cot x\|$
$1/\sqrt{(1 - x^2)}$	$\sin^{-1}x$	$1/(1 + x^2)$	$\tan^{-1}x$

Add + C to every indefinite integral.

03 Integration by substitution

Put the inner function = t so that its derivative is also present. $\int f(g(x))g'(x)dx = \int f(t)dt$.

- $\int 2x \sin(x^2)\,dx$: $t = x^2 \Rightarrow -\cos(x^2) + C$.
- $\int f'(x)/f(x)\,dx = \log|f(x)| + C$.
- Trig powers: use identities ($\sin^2 x = (1 - \cos 2x)/2$, $\sin^3 x = (3 \sin x - \sin 3x)/4$, product → sum).

04 Partial fractions

Denominator factor	Partial fraction
$(x - a)(x - b)$	$A/(x - a) + B/(x - b)$
$(x - a)^2$	$A/(x - a) + B/(x - a)^2$
$(x - a)(x^2 + bx + c)$	$A/(x - a) + (Bx + C)/(x^2 + bx + c)$

If the degree of numerator ≥ denominator, divide first.

05 Integration by parts

$$\int u\,v\,dx = u\int v\,dx - \int[u' \int v\,dx]\,dx$$

Choose u by **ILATE**: Inverse trig, Log, Algebraic, Trig, Exponential (first in list = u). $\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C$.

Example: $\int x e^x\,dx = x e^x - e^x + C$. $\int \log x\,dx = x \log x - x + C$.

06 Special integrals

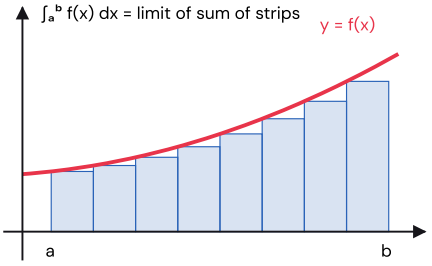
$\int$	**Result**
$dx/(x^2 - a^2)$	$(1/2a) \log\lvert(x - a)/(x + a)\rvert$
$dx/(a^2 - x^2)$	$(1/2a) \log\lvert(a + x)/(a - x)\rvert$
$dx/(x^2 + a^2)$	$(1/a) \tan^{-1}(x/a)$
$dx/\sqrt{(x^2 - a^2)}$	$\log\lvert x + \sqrt{(x^2 - a^2)}\rvert$
$dx/\sqrt{(a^2 - x^2)}$	$\sin^{-1}(x/a)$
$dx/\sqrt{(x^2 + a^2)}$	$\log\lvert x + \sqrt{(x^2 + a^2)}\rvert$
$\sqrt{(a^2 - x^2)}\,dx$	$(x/2)\sqrt{(a^2 - x^2)} + (a^2/2) \sin^{-1}(x/a)$
$\sqrt{(x^2 \pm a^2)}\,dx$	$(x/2)\sqrt{(x^2 \pm a^2)} \pm (a^2/2) \log\lvert x + \sqrt{(x^2 \pm a^2)}\rvert$

For $ax^2 + bx + c$: complete the square. For $(px + q)/(ax^2 + bx + c)$: write $px + q = A \cdot d/dx(ax^2 + bx + c) + B$.

07 Definite integrals & properties

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Second fundamental theorem. No + C needed. Area under the curve between a and b (with sign).



Definite integral as a limit of sums

Property	Statement
P1	$\int_a^b f = -\int_b^a f$; $\int_a^a f = 0$
P2	$\int_a^b f = \int_a^c f + \int_c^b f$
P3 (king)	$\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$
P4	$\int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx$
P5	$\int_{-a}^a f = 2\int_0^a f$ if f even; 0 if f odd
P6	$\int_0^{2a} f = 2\int_0^a f$ if $f(2a - x) = f(x)$; 0 if $f(2a - x) = -f(x)$

08 JEE Extra + Tricks

JEE LEVEL

- $\int e^x[f(x) + f'(x)]dx = e^x f(x) + C$ (very common).
- $\int dx/(a + b\cos x)$ and similar: use $t = \tan(x/2)$.
- King property: $\int_0^a f(x)dx = \int_0^a f(a - x)dx \Rightarrow \int_0^{\pi/2} \sin^n x/(\sin^n x + \cos^n x)\, dx = \pi/4$.
- $\int_{-a}^{a} f = 2\int_0^a f$ (even), 0 (odd); periodic: $\int_0^{nT} f = n\int_0^T f$.
- Walli's formula for $\int_0^{\pi/2} \sin^n x\, dx$: $[(n - 1)(n - 3)\ldots]/[n(n - 2)\ldots] \times (\pi/2$ if n even).
- ILATE order for by parts: Inverse, Log, Algebraic, Trig, Exponential (first function).
- Leibniz rule: $d/dx \int_{g(x)}^{h(x)} f(t)dt = f(h)h' - f(g)g'$.

09 Solved examples

Q1. $\int (x^3 + 5x^2 - 4)/x^2\, dx$.

$\int(x + 5 - 4x^{-2})\, dx = x^2/2 + 5x + 4/x + C$.

Q2. $\int dx/((x + 1)(x + 2))$.

$1/(x+1) - 1/(x+2) \Rightarrow \log|(x + 1)/(x + 2)| + C$.

Q3. $\int x \cos x\, dx$.

$u = x$: $x \sin x - \int \sin x\, dx = x \sin x + \cos x + C$.

Q4. $\int_0^{\pi/2} \sqrt{(\sin x)}/(\sqrt{(\sin x)} + \sqrt{(\cos x)})\, dx$.

By P4, $I = \int \sqrt{(\cos x)}/(\sqrt{\cos} + \sqrt{\sin}) \Rightarrow 2I = \int_0^{\pi/2} 1\, dx = \pi/2 \Rightarrow$ **$I = \pi/4$**.

Q5. Find $\int_{-1}^{1} x^5 \cos x\, dx$ and $\int dx/(x^2 + 2x + 2)$.

Odd function $\Rightarrow$ 0. Second: $(x + 1)^2 + 1 \Rightarrow \tan^{-1}(x + 1) + C$.

10 Common mistakes

- Forgetting + C in indefinite integrals.
- Not changing limits after substitution in definite integrals.
- Wrong choice of u in by parts (follow ILATE).
- $\int 1/x\,dx = \log x$ (should be $\log|x|$).

11 Practice questions

1. $\int \sin^2 x\,dx$.
2. $\int e^x(\sin x + \cos x)\,dx$.
3. $\int_0^1 x e^x\,dx$.
4. $\int_0^{\pi/2} \sin^2 x\,dx$.

ANSWERS

1) $x/2 - \sin 2x/4 + C$ · 2) $e^x \sin x + C$ · 3) 1 · 4) $\pi/4$

12 Quick revision: 5 minutes before the exam

METHODS

- Substitution: inner = t.
- Partial fractions for rationals.
- Parts: ILATE.

KEY RESULTS

- $f'/f \Rightarrow \log|f|$.
- $e^x(f + f') \Rightarrow e^x f$.

SPECIAL

- $1/(x^2+a^2) \Rightarrow (1/a)\tan^{-1}(x/a)$.
- Complete the square.

DEFINITE

- $F(b) - F(a)$.
- $a + b - x$ trick; odd $\Rightarrow 0$.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.