FREE NOTES · NCERT + JEE LEVEL

Application of Derivatives

Quantities ke badalne ki rate, increasing aur decreasing functions, first aur second derivative test se local maxima aur minima, closed interval pe absolute maxima-minima, aur optimisation ke word problems.

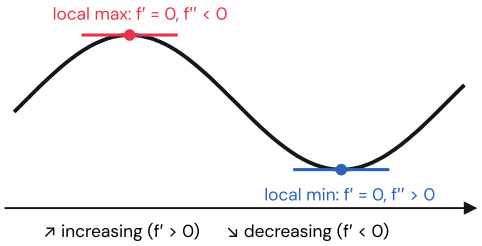
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01 Rate of change

dy/dx = x ke respect me y ke badalne ki rate. Dono time pe depend karein to: dy/dt = (dy/dx)(dx/dt).

Circle: $A = \pi r^2 \Rightarrow dA/dt = 2\pi r\, dr/dt$. Sphere: $V = (4/3)\pi r^3 \Rightarrow dV/dt = 4\pi r^2\, dr/dt$. Cube: $V = x^3 \Rightarrow dV/dt = 3x^2\, dx/dt$. Marginal cost = dC/dx, marginal revenue = dR/dx.

02 Increasing & decreasing functions



f′ ka sign batata upar ya neeche; extremes wahan jahan f′ = 0

Interval pe	f hai
f′(x) > 0	Strictly increasing
f′(x) < 0	Strictly decreasing
f′(x) = 0	Constant

Tareeka: f′ nikalo, f′ = 0 solve karo, number line ko in points pe todo, har hisse me f′ ka sign check karo.

03 Critical points

Wo points jahan f′(c) = 0 ya f′ exist na kare. Local maxima aur minima sirf critical points pe ho sakte (open interval pe defined function ke liye).

04 First derivative test

c paar karte waqt f′ ka sign	Nateeja
+ se −	Local maximum
− se +	Local minimum
Badlav nahi	Point of inflection (dono nahi)

05 Second derivative test

$f'(c) = 0$ aur $f''(c) < 0 \Rightarrow$ local maximum

$f'(c) = 0$ aur $f''(c) > 0 \Rightarrow$ local minimum

$f''(c) = 0 \Rightarrow$ test fail, first derivative test lagao

06 [a, b] pe absolute maximum & minimum

STEPS

1. (a, b) me saare critical points nikalo.
2. In points aur end points a, b pe f ki value nikalo.
3. Sabse badi value absolute max; sabse chhoti absolute min.

07 Optimisation (word problems)

1. Kya optimise karna hai (area, volume, cost…) pehchano aur di gayi shart se use ek variable ka function banao.
2. Differentiate karke 0 rakho.
3. Second derivative se max/min confirm karo.
4. Jo poocha wo answer do (dimensions, value).

Standard results: diye perimeter wala rectangle square hone pe max area; sphere me cylinder, sphere me cone (sphere radius r ho to h = 4r/3), square sheet se khula dabba (x = side/6 kaato).

JEE extra: (x_1, y_1) pe tangent: $y - y_1 = f'(x_1)(x - x_1)$; normal: slope $-1/f'(x_1)$.

08 JEE Extra + Tricks

JEE LEVEL

- (x_1, y_1) pe tangent: $y - y_1 = m(x - x_1)$; normal ka slope $-1/m$. Subtangent y/m, subnormal $y \cdot m$.
- Interval pe $f' \geq 0 \Rightarrow$ increasing; critical points jahan $f' = 0$ ya undefined.
- Second derivative test: $f'' < 0 \Rightarrow$ local max, $f'' > 0 \Rightarrow$ local min; $f'' = 0$ ho to first derivative test.
- Max/min trick: fixed perimeter pe rectangle ka area square me max; fixed surface pe cylinder ka volume $h = 2r$ pe max.
- Approximation: $\Delta y \approx f'(x)\Delta x$ (jaise $\sqrt{26} \approx 5 + 1/10$).

09 Solved examples

Q1. Circle ka radius 0.7 cm/s se badh raha. Circumference badhne ki rate?

$C = 2\pi r \Rightarrow dC/dt = 2\pi \times 0.7 =$ **1.4π cm/s**.

Q2. $f(x) = 2x^3 - 3x^2 - 36x + 7$ kahan increasing/decreasing?

$f' = 6(x - 3)(x + 2)$. Increasing $(-\infty, -2) \cup (3, \infty)$ pe, decreasing $(-2, 3)$ pe.

Q3. $f(x) = x^3 - 6x^2 + 9x + 15$ ke local max/min.

$f' = 3(x - 1)(x - 3)$; $f'' = 6x - 12$. $x = 1$: $f'' < 0 \Rightarrow$ max $f(1) = 19$. $x = 3$: $f'' > 0 \Rightarrow$ min $f(3) = 15$.

Q4. Jod 15 aur guna maximum wale do positive numbers?

$P = x(15 - x)$, $P' = 15 - 2x = 0 \Rightarrow x = 7.5 \Rightarrow$ numbers **7.5 aur 7.5**.

Q5. $[1, 5]$ pe $f(x) = 2x^3 - 15x^2 + 36x + 1$ ke absolute max/min.

$f' = 6(x - 2)(x - 3)$. $f(1) = 24$, $f(2) = 29$, $f(3) = 28$, $f(5) = 56 \Rightarrow$ max **56** ($x = 5$), min **24** ($x = 1$).

10 Common mistakes

- Absolute extrema me end points bhool jaana.
- Har critical point ko max ya min kehna.
- Differentiate karne se pehle function ko ek variable me na likhna.
- Intervals me f' ka sign galat (har hisse me ek value daal ke check karo).

11 Practice questions

1. Kya $f(x) = e^x$ R pe increasing hai?
2. $x^2 - 8x + 17$ ki minimum value?
3. Sphere ka volume 10 cm^3/s se badh raha. $r = 5$ cm pe radius kitni tezi se?
4. Perimeter 40 wale rectangle ka max area.

ANSWERS

1) Haan ($f' = e^x > 0$) · 2) $x = 4$ pe 1 · 3) $1/(10\pi)$ cm/s · 4) 100 (side 10 ka square)

12 Quick revision: exam se 5 minute pehle

RATES

- dy/dt = (dy/dx)(dx/dt).
- Sphere dV/dt = 4πr² dr/dt.

MONOTONIC

- f′ > 0 increasing.
- f′ < 0 decreasing.

EXTREMA

- f′ = 0 phir f″ < 0 max, > 0 min.
- Sign + → − ⇒ max.

ABSOLUTE

- Critical points + ends check.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.