FREE NOTES · NCERT + JEE LEVEL

Continuity & Differentiability

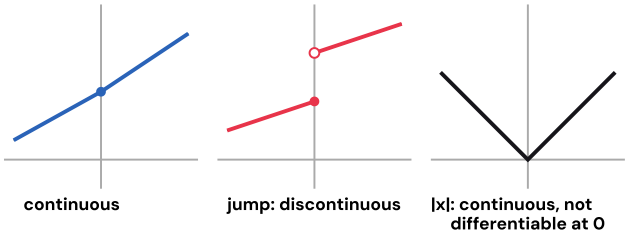
Point aur interval pe continuity, differentiability, chain rule, implicit functions, inverse trigonometric, exponential aur logarithmic derivatives, logarithmic differentiation, parametric forms aur second-order derivatives.

01 Continuity
02 Differentiability
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01 Continuity

$$f, a \text{ pe continuous} \Leftrightarrow \lim_{x \to a^-} f = \lim_{x \to a^+} f = f(a)$$

Function continuous tab jab domain ke har point pe continuous ho. Polynomials, sin, cos, e^x, $|x|$ har jagah continuous; rational functions, tan, log apne domain pe. Continuous functions ka jod, fark, guna, bhaag (denominator $\neq 0$) aur composition bhi continuous.



Continuous, jump discontinuity, aur kona (corner)

02 Differentiability

$\text{LHD} = \lim_{h \to 0^-} [f(a+h) - f(a)]/h, \ \text{RHD} = \lim_{h \to 0^+} [f(a+h) - f(a)]/h$

a pe differentiable ⇔ LHD = RHD (finite).

ZAROORI BAAT

Differentiable ⇒ continuous, par continuous ⇏ differentiable. 0 pe |x| aur integers pe [x] standard examples (|x| me kona; [x] continuous bhi nahi).

03 Chain rule

$$dy/dx = (dy/du) \cdot (du/dx)$$

Bahar se andar differentiate karo. d/dx sin(x²) = cos(x²)·2x. d/dx $(2x + 3)^5 = 5(2x + 3)^4 \cdot 2$.

04 Implicit functions

Jab y alag na ho ($x^2 + y^2 = 25$), dono taraf x ke respect me differentiate karo, y ko x ka function maan ke (d/dx $y^2 = 2y\, dy/dx$), phir dy/dx nikalo. Yahan $2x + 2y\, y' = 0 \Rightarrow dy/dx = -x/y$.

05 Standard derivatives

f(x)	f′(x)	f(x)	f′(x)
$\sin^{-1}x$	$1/\sqrt{(1-x^2)}$	$\cos^{-1}x$	$-1/\sqrt{(1-x^2)}$
$\tan^{-1}x$	$1/(1+x^2)$	$\cot^{-1}x$	$-1/(1+x^2)$
$\sec^{-1}x$	$1/(\lvert x\rvert\sqrt{(x^2-1)})$	$\operatorname{cosec}^{-1}x$	$-1/(\lvert x\rvert\sqrt{(x^2-1)})$
e^x	e^x	a^x	$a^x \log a$
$\log x$	$1/x$	$\log_a x$	$1/(x \log a)$

06 Logarithmic differentiation

y = [u(x)]^{v(x)} ya lambe guna/bhaag ke liye: log lo, phir differentiate karo.

$$y = x^x \Rightarrow \log y = x \log x \Rightarrow (1/y)\, y' = \log x + 1 \Rightarrow y' = x^x(1 + \log x)$$

07 Parametric & second-order derivatives

$$x = f(t),\ y = g(t) \Rightarrow dy/dx = (dy/dt)/(dx/dt)$$

$d^2y/dx^2 = d/dx(dy/dx)$. Parametric me: $d^2y/dx^2 = [d/dt(dy/dx)] / (dx/dt)$

Example: x = a cos θ, y = a sin θ ⇒ dy/dx = −cot θ.

JEE extra: Rolle's theorem aur Mean Value Theorem (f′(c) = [f(b) − f(a)]/(b − a)).

08 JEE Extra + Tricks

JEE LEVEL

- Differentiability ⇒ continuity (ulta nahi): $|x|$ 0 pe continuous par differentiable nahi.
- $y = f(x)^{g(x)}$ ke liye logarithmic differentiation: ln y = g ln f.
- Parametric: $dy/dx = (dy/dt)/(dx/dt)$; $d^2y/dx^2 = d/dt(dy/dx) \div dx/dt$.
- Rolle aur LMV theorems (JEE): $f'(c) = 0$ / $f'(c) = (f(b) - f(a))/(b - a)$.
- Inverse trig ke liye pehle $x = \tan\theta$, $\sin\theta$, $\cos\theta$ rakhkar saral karo.

09 Solved examples

Q1. $f(x) = kx^2$ $(x \le 2)$, 3 $(x > 2)$ ko 2 pe continuous banane ke liye k.

$4k = 3 \Rightarrow$ **$k = 3/4$**.

Q2. $\sin(\cos(x^2))$ differentiate karo.

$\cos(\cos x^2)\cdot(-\sin x^2)\cdot 2x =$ **$-2x \sin(x^2) \cos(\cos x^2)$**.

Q3. $x^2 + xy + y^2 = 100$ ho to dy/dx.

$2x + y + xy' + 2yy' = 0 \Rightarrow$ **$y' = -(2x + y)/(x + 2y)$**.

Q4. $y = \sin^{-1}(2x\sqrt{(1 - x^2)})$ ho to dy/dx $(-1/\sqrt{2} < x < 1/\sqrt{2})$.

$x = \sin\theta \Rightarrow y = 2\theta = 2\sin^{-1}x \Rightarrow$ **$2/\sqrt{(1 - x^2)}$**.

Q5. $y = 3e^{2x} + 2e^{3x}$ ho to dikhao $y'' - 5y' + 6y = 0$.

$y' = 6e^{2x} + 6e^{3x}$, $y'' = 12e^{2x} + 18e^{3x} \Rightarrow 12 - 30 + 18 = 0$ aur $18 - 30 + 12 = 0$ ✓.

10 Common mistakes

- Chain rule me andar wala derivative bhool jaana.
- $d/dx(x^x) = x\cdot x^{x-1}$ likhna (power rule nahi lagta).
- Kehna ki continuity ⇒ differentiability.
- Parametric second derivative me dx/dt se bhaag na dena.

11 Practice questions

1. Kya $f(x) = |x - 3|$, 3 pe differentiable hai?
2. $d/dx\,(\log(\log x))$.
3. $d/dx\,(\tan^{-1}((1 + x)/(1 - x)))$?
4. $x = at^2$, $y = 2at$ ho to dy/dx.

ANSWERS

1) Nahi (LHD −1, RHD 1) · 2) $1/(x \log x)$ · 3) $1/(1 + x^2)$ · 4) $1/t$

12 Quick revision: exam se 5 minute pehle

CONTINUITY

- LHL = RHL = f(a).
- Differentiable ⇒ continuous.

RULES

- Chain: bahar → andar.
- Implicit: har y term ke saath y′.

INVERSE TRIG

- $\sin^{-1}$: $1/\sqrt{(1-x^2)}$.
- $\tan^{-1}$: $1/(1+x^2)$.

SPECIAL

- u^v ⇒ log lo.
- Parametric: (dy/dt)/(dx/dt).

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.