FREE NOTES · NCERT + JEE LEVEL

Continuity & Differentiability

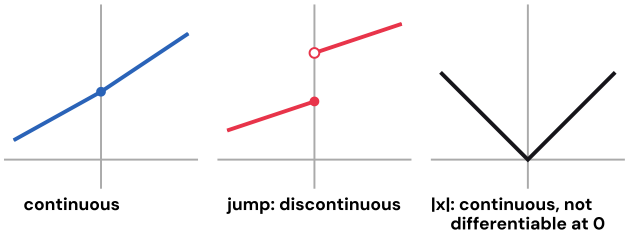
Continuity at a point and on an interval, differentiability, chain rule, implicit functions, inverse trigonometric, exponential and logarithmic derivatives, logarithmic differentiation, parametric forms and second-order derivatives.

01 Continuity
02 Differentiability
03 Chain rule
04 Implicit functions
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01 Continuity

$$f \text{ continuous at } a \iff \lim_{x \to a^-} f = \lim_{x \to a^+} f = f(a)$$

A function is continuous if it's continuous at every point of its domain. Polynomials, sin, cos, e^x, $|x|$ are continuous everywhere; rational functions, tan, log are continuous on their domains. Sum, difference, product, quotient (denominator ≠ 0) and composition of continuous functions are continuous.



Continuous, jump discontinuity, and a corner

02 Differentiability

$$\text{LHD} = \lim_{h \to 0^-} [f(a+h) - f(a)]/h, \quad \text{RHD} = \lim_{h \to 0^+} [f(a+h) - f(a)]/h$$

Differentiable at a ⟺ LHD = RHD (finite).

KEY FACT

Differentiable ⇒ continuous, but continuous ⇏ differentiable. |x| at 0 and [x] at integers are the standard examples (|x| has a corner; [x] is not even continuous).

03 Chain rule

$$dy/dx = (dy/du) \cdot (du/dx)$$

Differentiate outside to inside. d/dx sin(x²) = cos(x²)·2x. d/dx $(2x + 3)^5$ = $5(2x + 3)^4 \cdot 2$.

04 Implicit functions

When y is not separated ($x^2 + y^2 = 25$), differentiate both sides w.r.t. x, treating y as a function of x (d/dx y^2 = 2y dy/dx), then solve for dy/dx. Here 2x + 2y y′ = 0 ⇒ dy/dx = −x/y.

05 Standard derivatives

f(x)	f′(x)	f(x)	f′(x)
$\sin^{-1}x$	$1/\sqrt{(1-x^2)}$	$\cos^{-1}x$	$-1/\sqrt{(1-x^2)}$
$\tan^{-1}x$	$1/(1+x^2)$	$\cot^{-1}x$	$-1/(1+x^2)$
$\sec^{-1}x$	$1/(\lvert x\rvert\sqrt{(x^2-1)})$	$\operatorname{cosec}^{-1}x$	$-1/(\lvert x\rvert\sqrt{(x^2-1)})$
e^x	e^x	a^x	$a^x \log a$
$\log x$	$1/x$	$\log_a x$	$1/(x \log a)$

06 Logarithmic differentiation

For y = [u(x)]^{v(x)} or long products/quotients: take log, then differentiate.

$$y = x^x \Rightarrow \log y = x \log x \Rightarrow (1/y)\, y' = \log x + 1 \Rightarrow y' = x^x(1 + \log x)$$

07 Parametric & second-order derivatives

$$x = f(t),\ y = g(t) \Rightarrow dy/dx = (dy/dt)/(dx/dt)$$

$d^2y/dx^2 = d/dx(dy/dx)$. For parametric: $d^2y/dx^2 = [d/dt(dy/dx)] / (dx/dt)$

Example: x = a cos θ, y = a sin θ ⇒ dy/dx = −cot θ.

JEE extra: Rolle's theorem and the Mean Value Theorem (f′(c) = [f(b) − f(a)]/(b − a)).

08 JEE Extra + Tricks

JEE LEVEL

- Differentiability ⇒ continuity (not converse): $|x|$ continuous but not differentiable at 0.
- Logarithmic differentiation for $y = f(x)^{g(x)}$: $\ln y = g \ln f$.
- Parametric: $dy/dx = (dy/dt)/(dx/dt)$; $d^2y/dx^2 = d/dt(dy/dx) \div dx/dt$.
- Rolle's and LMV theorems (JEE): $f'(c) = 0$ / $f'(c) = (f(b) - f(a))/(b - a)$.
- Substitution tricks for inverse trig: $x = \tan\theta$, $\sin\theta$, $\cos\theta$ to simplify before differentiating.

09 Solved examples

Q1. Find k so that $f(x) = kx^2$ $(x \le 2)$, 3 $(x > 2)$ is continuous at 2.

$4k = 3 \Rightarrow$ **$k = 3/4$**.

Q2. Differentiate $\sin(\cos(x^2))$.

$\cos(\cos x^2)\cdot(-\sin x^2)\cdot 2x =$ **$-2x \sin(x^2) \cos(\cos x^2)$**.

Q3. Find dy/dx if $x^2 + xy + y^2 = 100$.

$2x + y + xy' + 2yy' = 0 \Rightarrow$ **$y' = -(2x + y)/(x + 2y)$**.

Q4. If $y = \sin^{-1}(2x\sqrt{(1 - x^2)})$, find dy/dx $(-1/\sqrt{2} < x < 1/\sqrt{2})$.

$x = \sin\theta \Rightarrow y = 2\theta = 2\sin^{-1}x \Rightarrow$ **$2/\sqrt{(1 - x^2)}$**.

Q5. If $y = 3e^{2x} + 2e^{3x}$, show $y'' - 5y' + 6y = 0$.

$y' = 6e^{2x} + 6e^{3x}$, $y'' = 12e^{2x} + 18e^{3x} \Rightarrow 12 - 30 + 18 = 0$ and $18 - 30 + 12 = 0$ ✓.

10 Common mistakes

- Forgetting the inner derivative in the chain rule.
- Writing $d/dx(x^x) = x\cdot x^{x-1}$ (power rule doesn't apply).
- Saying continuity ⇒ differentiability.
- In parametric second derivative, forgetting to divide by dx/dt.

11 Practice questions

1. Is $f(x) = |x - 3|$ differentiable at 3?
2. $d/dx\ (\log(\log x))$.
3. $d/dx\ (\tan^{-1}((1 + x)/(1 - x)))$?
4. If $x = at^2$, $y = 2at$, find dy/dx.

ANSWERS

1) No (LHD −1, RHD 1) · 2) $1/(x \log x)$ · 3) $1/(1 + x^2)$ · 4) $1/t$

12 Quick revision: 5 minutes before the exam

CONTINUITY

- LHL = RHL = f(a).
- Differentiable ⇒ continuous.

RULES

- Chain: outside → inside.
- Implicit: y′ with every y term.

INVERSE TRIG

- $\sin^{-1}$: $1/\sqrt{(1-x^2)}$.
- $\tan^{-1}$: $1/(1+x^2)$.

SPECIAL

- u^v ⇒ take log.
- Parametric: (dy/dt)/(dx/dt).

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.