FREE NOTES · NCERT + JEE LEVEL

Determinants

2×2 aur 3×3 matrices ka determinant, triangle ka area, minors aur cofactors, matrix ka adjoint aur inverse, aur consistency test ke saath matrix method se linear equations solve karna.

01 Order 2 aur 3 ka determinant
02 Kaam ki properties
03 Triangle ka area
04 Minors & cofactors
05 Adjoint & inverse
06 Linear equations solve karna (matrix method)
07 Consistency test
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09 Solved examples
10 Common mistakes
11 Practice questions
12 Quick revision: exam se 5 minute pehle

01 Order 2 aur 3 ka determinant

$$|a\ b;\ c\ d| = ad - bc$$

$$|A|\ (3\times3) = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Sirf square matrices ka hota. Sign pattern ke saath kisi bhi row ya column se expand kar sakte; jisme zyada zeros ho wo lo.

02 Kaam ki properties

- $|A'| = |A|$. Do rows badlo to sign badal jaata. Do rows same ⇒ $|A| = 0$.
- Ek row ko k se guna ⇒ $|A|$ bhi k guna; isliye n×n ke liye **$|kA| = k^n|A|$**.
- Ek row me doosri row ka multiple jodne se $|A|$ nahi badalta.
- $|AB| = |A||B|$; $|A^{-1}| = 1/|A|$; $|\text{adj } A| = |A|^{n-1}$.

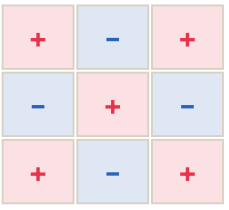
03 Triangle ka area

$$\Delta = \tfrac{1}{2}\,|x_1\ y_1\ 1;\ x_2\ y_2\ 1;\ x_3\ y_3\ 1|$$

Absolute value lo. Area = 0 ⇔ teeno points **collinear**. (x_1, y_1), (x_2, y_2) se line ki equation: $|x\ y\ 1;\ x_1\ y_1\ 1;\ x_2\ y_2\ 1| = 0$.

04 Minors & cofactors

- **Minor** M_{ij} = row i aur column j hata ke bacha determinant.
- **Cofactor** $A_{ij} = (-1)^{i+j} M_{ij}$.
- |A| = kisi row ke (elements × unke apne cofactors) ka jod.
- Ek row ke elements × **doosri** row ke cofactors = 0.



Sign pattern $(-1)^{i+j}$

05 Adjoint & inverse

$\text{adj } A = \text{cofactor matrix ka transpose}$

$A(\text{adj } A) = (\text{adj } A)A = |A|\, I$

$$A^{-1} = (1/|A|)\ \text{adj } A,\ |A| \neq 0$$

|A| = 0 ⇒ **singular** (inverse nahi). 2×2 shortcut: adj [[a, b], [c, d]] = [[d, −b], [−c, a]].

06 Linear equations solve karna (matrix method)

AX = B likho. |A| ≠ 0 ho to: **X = A⁻¹B** (unique solution).

07 Consistency test

Case	Nateeja
|A| ≠ 0	Consistent, unique solution
|A| = 0 aur (adj A)B ≠ O	Inconsistent, koi solution nahi
|A| = 0 aur (adj A)B = O	Infinite solutions ke saath consistent (ya koi nahi; check karo)

08 JEE Extra + Tricks

JEE LEVEL

- $|kA| = k^n|A|$ (n = order); $|\text{adj } A| = |A|^{n-1}$; $\text{adj}(\text{adj } A) = |A|^{n-2}A$; $|A^{-1}| = 1/|A|$.
- Properties: rows badalne se sign badalta; do same rows $\Rightarrow 0$; $R_i \to R_i + kR_j$ se nahi badalta.
- Cramer: $D \neq 0$ unique solution; $D = 0$ aur sab $D_x, D_y, D_z = 0 \Rightarrow$ infinite ya koi nahi (check); $D = 0$, koi $D_i \neq 0 \Rightarrow$ solution nahi.
- Homogeneous system ka non-trivial solution $\Leftrightarrow D = 0$.
- Triangle area = ½|determinant|; collinear $\Rightarrow 0$.

09 Solved examples

Q1. |1 2 4; −1 3 0; 4 1 0| nikalo.

C3 se expand: 4 × (−1 − 12) = **−52**.

Q2. Vertices (1, 0), (6, 0), (4, 3) wale triangle ka area.

½|1(0 − 3) − 0 + 1(18 − 0)| = ½ × 15 = **7.5** sq units.

Q3. A = [[2, 3], [1, 4]] ka A^{-1}.

|A| = 5, adj A = [[4, −3], [−1, 2]] ⇒ A^{-1} = (1/5)[[4, −3], [−1, 2]].

Q4. 2x + 5y = 1, 3x + 2y = 7 solve karo.

|A| = −11; X = $A^{-1}B$ = (−1/11)[[2, −5], [−3, 2]][1, 7]^T = (−1/11)[−33, 11] ⇒ **x = 3, y = −1**.

10 Common mistakes

- |kA| = k|A| likhna (k^n|A| hai).
- adj A ke liye cofactor matrix ka transpose bhool jaana.
- Cofactor ke galat signs.
- Area negative chhod dena.

11 Practice questions

1. 3×3 matrix ka |A| = 3 ho to |2A| aur |adj A|.
2. (k, 0), (4, 0), (0, 2) wale triangle ka area 4 ho to k.
3. Kya x + y = 2, 2x + 2y = 5 consistent hai?
4. [[1, 2], [3, 4]] ka cofactor A_{12}?

ANSWERS

1) 24, 9 · 2) k = 0 ya 8 · 3) Nahi · 4) −3

12 Quick revision: exam se 5 minute pehle

VALUES

- ad − bc.
- Zeros wali row se expand.

PROPERTIES

- $|kA| = k^n|A|$.
- $|AB| = |A||B|$.
- $|\text{adj } A| = |A|^{n-1}$.

INVERSE

- $A^{-1} = \text{adj } A/|A|$.
- $|A| = 0 \Rightarrow$ singular.

EQUATIONS

- $X = A^{-1}B$.
- $|A| = 0$: (adj A)B check.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.