FREE NOTES · NCERT + JEE LEVEL

Determinants

Determinant of 2×2 and 3×3 matrices, area of a triangle, minors and cofactors, adjoint and inverse of a matrix, and solving linear equations by the matrix method with the consistency test.

01 Determinant of order 2 and 3
02 Useful properties
03 Area of a triangle
04 Minors & cofactors
05 Adjoint & inverse
06 Solving linear equations (matrix method)
07 Consistency test
08 JEE Extra + Tricks
09 Solved examples
10 Common mistakes
11 Practice questions
12 Quick revision

01 Determinant of order 2 and 3

$$|a\ b;\ c\ d| = ad - bc$$

$$|A|\ (3\times3) = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Defined only for square matrices. You can expand along any row or column using the sign pattern; choose the one with the most zeros.

02 Useful properties

- $|A'| = |A|$. Swapping two rows changes the sign. Two identical rows ⇒ $|A| = 0$.
- Multiplying one row by k multiplies $|A|$ by k; so **$|kA| = k^n|A|$** for n×n.
- Adding a multiple of one row to another doesn't change $|A|$.
- $|AB| = |A||B|$; $|A^{-1}| = 1/|A|$; $|\text{adj } A| = |A|^{n-1}$.

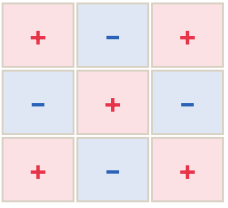
03 Area of a triangle

$$\Delta = \tfrac{1}{2}\,|x_1\ y_1\ 1;\ x_2\ y_2\ 1;\ x_3\ y_3\ 1|$$

Take the absolute value. Area = 0 ⇔ the three points are **collinear**. Equation of the line through (x_1, y_1), (x_2, y_2): $|x\ y\ 1;\ x_1\ y_1\ 1;\ x_2\ y_2\ 1| = 0$.

04 Minors & cofactors

- **Minor** M_{ij} = determinant left after deleting row i and column j.
- **Cofactor** $A_{ij} = (-1)^{i+j} M_{ij}$.
- |A| = sum of (elements of any row × their own cofactors).
- Elements of a row × cofactors of **another** row = 0.



Sign pattern $(-1)^{i+j}$

05 Adjoint & inverse

$\text{adj } A =$ transpose of the cofactor matrix

$A(\text{adj } A) = (\text{adj } A)A = |A|\, I$

$$A^{-1} = (1/|A|)\ \text{adj } A,\ |A| \neq 0$$

$|A| = 0 \Rightarrow$ **singular** (no inverse). 2×2 shortcut: adj [[a, b], [c, d]] = [[d, −b], [−c, a]].

06 Solving linear equations (matrix method)

Write AX = B. If $|A| \neq 0$: $\mathbf{X = A^{-1}B}$ (unique solution).

07 Consistency test

Case	Result
$\|A\| \neq 0$	Consistent, unique solution
$\|A\| = 0$ and $(\text{adj } A)B \neq O$	Inconsistent, no solution
$\|A\| = 0$ and $(\text{adj } A)B = O$	Consistent with infinitely many solutions (or none; check)

08 JEE Extra + Tricks

JEE LEVEL

- $|kA| = k^n|A|$ (n = order); $|\text{adj } A| = |A|^{n-1}$; $\text{adj}(\text{adj } A) = |A|^{n-2}A$; $|A^{-1}| = 1/|A|$.
- Properties: swapping rows changes sign; two identical rows ⇒ 0; $R_i \to R_i + kR_j$ unchanged.
- Cramer's rule: $D \neq 0$ unique solution; $D = 0$ and all $D_x, D_y, D_z = 0$ ⇒ infinite or none (check); $D = 0$, any $D_i \neq 0$ ⇒ no solution.
- Homogeneous system has non-trivial solution iff $D = 0$.
- Area of triangle = ½|determinant|; collinear points ⇒ 0.

09 Solved examples

Q1. Evaluate $|1\ 2\ 4; -1\ 3\ 0; 4\ 1\ 0|$.

Expand along C3: $4 \times (-1 - 12) = \mathbf{-52}$.

Q2. Area of the triangle with vertices (1, 0), (6, 0), (4, 3).

$\frac{1}{2}|1(0 - 3) - 0 + 1(18 - 0)| = \frac{1}{2} \times 15 = \mathbf{7.5}$ sq units.

Q3. Find A^{-1} for $A = [[2, 3], [1, 4]]$.

$|A| = 5$, adj $A = [[4, -3], [-1, 2]] \Rightarrow A^{-1} = (1/5)[[4, -3], [-1, 2]]$.

Q4. Solve $2x + 5y = 1$, $3x + 2y = 7$.

$|A| = -11$; $X = A^{-1}B = (-1/11)[[2, -5], [-3, 2]][1, 7]^{T} = (-1/11)[-33, 11] \Rightarrow \mathbf{x = 3, y = -1}$.

10 Common mistakes

- Writing $|kA| = k|A|$ (it is $k^{n}|A|$).
- Forgetting to transpose the cofactor matrix for adj A.
- Using wrong cofactor signs.
- Leaving the area negative.

11 Practice questions

1. If $|A| = 3$ for a 3×3 matrix, find $|2A|$ and $|\text{adj } A|$.
2. Find k if the area of the triangle with vertices (k, 0), (4, 0), (0, 2) is 4.
3. Is the system $x + y = 2$, $2x + 2y = 5$ consistent?
4. Cofactor A_{12} of $[[1, 2], [3, 4]]$?

ANSWERS

1) 24, 9 · 2) k = 0 or 8 · 3) No · 4) −3

12 Quick revision: 5 minutes before the exam

VALUES

- ad − bc.
- Expand along zeros.

PROPERTIES

- $|kA| = k^n|A|$.
- $|AB| = |A||B|$.
- $|\text{adj } A| = |A|^{n-1}$.

INVERSE

- $A^{-1} = \text{adj } A/|A|$.
- $|A| = 0 \Rightarrow$ singular.

EQUATIONS

- $X = A^{-1}B$.
- $|A| = 0$: check (adj A)B.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.