FREE NOTES · NCERT + JEE LEVEL

Matrices

Matrix kya hai, order aur types, equality, addition, scalar multiplication, matrix multiplication aur uski properties, transpose, symmetric aur skew-symmetric matrices, aur invertible matrix ka idea.

01 Matrix & order
02 Matrices ke types
03 Addition & scalar multiplication
04 Matrix multiplication
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01 Matrix & order

Matrix numbers ka rectangular array hai. $A = [a_{ij}]_{m\times n}$ me m rows aur n columns; a_{ij} row i, column j me. Elements = mn. 12 elements wale matrix ka order 1×12, 12×1, 2×6, 6×2, 3×4, 4×3 ho sakta.

02 Matrices ke types

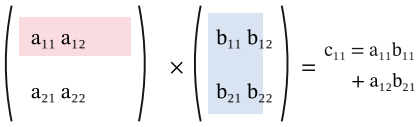
Type	Definition
Row / column	1 × n / m × 1
Square	m = n
Diagonal	Square, diagonal ke bahar sab 0
Scalar	Diagonal, saare diagonal elements barabar
Identity I	Diagonal elements 1
Zero O	Saare elements 0

Do matrices **barabar** agar order same aur corresponding elements barabar.

03 Addition & scalar multiplication

- A + B sirf same order pe: corresponding elements jodo. Commutative aur associative; A + O = A; A + (−A) = O.
- kA: har element ko k se guna. k(A + B) = kA + kB; (k + l)A = kA + lA.

04 Matrix multiplication



row of A × column of B, multiply pairwise and add

Element c_{ij} = (A ki row i) · (B ka column j)

$A_{m\times n}\, B_{n\times p} = C_{m\times p}$: A ke columns = B ki rows hone chahiye. $c_{ij} = \Sigma_k\, a_{ik}\, b_{kj}$.

PROPERTIES

- Associative: (AB)C = A(BC). Distributive: A(B + C) = AB + AC.
- **Commutative nahi**: aam taur pe AB ≠ BA (ek undefined bhi ho sakta).
- AB = O ka matlab A = O ya B = O nahi.
- AI = IA = A.

05 Transpose

A′ (ya A^T): rows columns ban jaati. Order m × n → n × m.

$(A')' = A;\ (kA)' = kA';\ (A + B)' = A' + B';\ \mathbf{(AB)' = B'A'}$

06 Symmetric & skew-symmetric matrices

- **Symmetric:** A′ = A ($a_{ij} = a_{ji}$).
- **Skew-symmetric:** A′ = −A ($a_{ij} = -a_{ji}$); saare diagonal elements 0.
- A + A′ symmetric; A − A′ skew-symmetric.

HAR SQUARE MATRIX KA UNIQUE SPLIT

$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = \text{symmetric} + \text{skew-symmetric}$

07 Invertible matrices

Square matrix A invertible agar B ho jisse AB = BA = I; tab B = A^{-1} (unique). $(AB)^{-1} = B^{-1}A^{-1}$. Adjoint se A^{-1} nikalna Determinants chapter me. (Elementary row operations wala method ab NCERT syllabus me nahi.)

08 JEE Extra + Tricks

JEE LEVEL

- Har square matrix = symmetric + skew-symmetric: $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$.
- Skew-symmetric ke diagonal elements 0; odd order ka determinant 0.
- $(AB)^T = B^T A^T$; $(AB)^{-1} = B^{-1}A^{-1}$; aam taur pe $AB \neq BA$.
- Idempotent $A^2 = A$; involutory $A^2 = I$; nilpotent $A^k = O$; orthogonal $AA^T = I$.
- Trace(A + B) = trace A + trace B; trace(AB) = trace(BA).

09 Solved examples

Q1. $a_{ij} = (i + j)^2/2$ wala 2×2 matrix banao.

$a_{11} = 2$, $a_{12} = 9/2$, $a_{21} = 9/2$, $a_{22} = 8$ ⇒ [[2, 9/2], [9/2, 8]].

Q2. A = [[1, 2], [3, 4]], B = [[2, 0], [1, 3]]. AB aur BA nikalo.

AB = [[4, 6], [10, 12]]; BA = [[2, 4], [10, 14]]. AB ≠ BA.

Q3. A = [[3, 1], [−1, 2]] ho to dikhao $A^2 - 5A + 7I = O$.

A^2 = [[8, 5], [−5, 3]]; 5A = [[15, 5], [−5, 10]]; 8 − 15 + 7 = 0, 5 − 5 = 0, −5 + 5 = 0, 3 − 10 + 7 = 0 ✓.

Q4. A = [[1, 5], [−1, 2]] ko symmetric + skew-symmetric me likho.

P = ½(A + A′) = [[1, 2], [2, 2]]; Q = ½(A − A′) = [[0, 3], [−3, 0]].

10 Common mistakes

- AB = BA maan ke $(A + B)^2 = A^2 + 2AB + B^2$ likhna (asal me $A^2 + AB + BA + B^2$).
- $(AB)' = A'B'$ likhna (order ulta).
- Order match na karne wale matrices ka guna.
- Kehna ki skew-symmetric ke diagonal me non-zero ho sakta.

11 Practice questions

1. A 2×3 aur B 3×4 ho to AB ka order?
2. x, y nikalo: [[x + y, 2], [5, x − y]] = [[6, 2], [5, 2]].
3. A symmetric aur B skew-symmetric ho to kya AB − BA symmetric hai?
4. 0 ya 1 entries wale kitne 2×2 matrices?

ANSWERS

1) 2×4 · 2) x = 4, y = 2 · 3) Haan · 4) 16

12 Quick revision: exam se 5 minute pehle

BASICS

- m × n: m rows.
- Barabar: same order + elements.

GUNA

- (m×n)(n×p) = m×p.
- Aam taur pe AB ≠ BA.

TRANSPOSE

- $(AB)' = B'A'$.
- A' = A sym; A' = −A skew.

SPLIT

- $A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$.
- $(AB)^{-1} = B^{-1}A^{-1}$.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.