FREE NOTES · NCERT + JEE LEVEL

Matrices

What a matrix is, order and types, equality, addition, scalar multiplication, matrix multiplication and its properties, transpose, symmetric and skew-symmetric matrices, and the idea of an invertible matrix.

01 Matrix & order
02 Types of matrices
03 Addition & scalar multiplication
04 Matrix multiplication
05 Transpose
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01 Matrix & order

A matrix is a rectangular array of numbers. $A = [a_{ij}]_{m\times n}$ has m rows and n columns; a_{ij} is in row i, column j. Number of elements = mn. A matrix with 12 elements can have order 1×12, 12×1, 2×6, 6×2, 3×4, 4×3.

02 Types of matrices

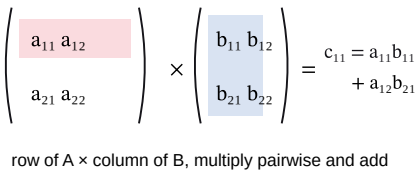
Type	Definition
Row / column	1 × n / m × 1
Square	m = n
Diagonal	Square, all off-diagonal elements 0
Scalar	Diagonal with all diagonal elements equal
Identity I	Diagonal elements 1
Zero O	All elements 0

Two matrices are **equal** if they have the same order and corresponding elements are equal.

03 Addition & scalar multiplication

- A + B defined only for same order: add corresponding elements. Commutative and associative; A + O = A; A + (−A) = O.
- kA: multiply every element by k. k(A + B) = kA + kB; (k + l)A = kA + lA.

04 Matrix multiplication



row of A × column of B, multiply pairwise and add

Element c_{ij} = (row i of A) · (column j of B)

$A_{m\times n}\, B_{n\times p} = C_{m\times p}$: columns of A must equal rows of B. $c_{ij} = \Sigma_k\, a_{ik}\, b_{kj}$.

PROPERTIES

- Associative: (AB)C = A(BC). Distributive: A(B + C) = AB + AC.
- **Not commutative**: AB ≠ BA in general (one may even be undefined).
- AB = O doesn't mean A = O or B = O.
- AI = IA = A.

05 Transpose

A' (or A^T): rows become columns. Order m × n → n × m.

$(A')' = A; (kA)' = kA'; (A + B)' = A' + B'; \mathbf{(AB)' = B'A'}$

06 Symmetric & skew-symmetric matrices

- **Symmetric:** $A' = A$ ($a_{ij} = a_{ji}$).
- **Skew-symmetric:** $A' = -A$ ($a_{ij} = -a_{ji}$); all diagonal elements are 0.
- $A + A'$ is symmetric; $A - A'$ is skew-symmetric.

EVERY SQUARE MATRIX SPLITS UNIQUELY

$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = \text{symmetric} + \text{skew-symmetric}$

07 Invertible matrices

A square matrix A is invertible if there is B with AB = BA = I; then $B = A^{-1}$ (unique). $(AB)^{-1} = B^{-1}A^{-1}$. Finding A^{-1} using adjoints is done in Determinants. (Elementary row operations method is no longer in the NCERT syllabus.)

08 JEE Extra + Tricks

JEE LEVEL

- Every square matrix = symmetric + skew-symmetric: $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$.
- Diagonal elements of a skew-symmetric matrix are 0; its determinant (odd order) is 0.
- $(AB)^T = B^T A^T$; $(AB)^{-1} = B^{-1}A^{-1}$; $AB \neq BA$ in general.
- Idempotent $A^2 = A$; involutory $A^2 = I$; nilpotent $A^k = O$; orthogonal $AA^T = I$.
- Trace$(A + B)$ = trace A + trace B; trace(AB) = trace(BA).

09 Solved examples

Q1. Construct a 2×2 matrix with $a_{ij} = (i + j)^2/2$.

$a_{11} = 2$, $a_{12} = 9/2$, $a_{21} = 9/2$, $a_{22} = 8 \Rightarrow [[2, 9/2], [9/2, 8]]$.

Q2. A = [[1, 2], [3, 4]], B = [[2, 0], [1, 3]]. Find AB and BA.

AB = [[4, 6], [10, 12]]; BA = [[2, 4], [10, 14]]. AB ≠ BA.

Q3. If A = [[3, 1], [−1, 2]], show $A^2 - 5A + 7I = O$.

A^2 = [[8, 5], [−5, 3]]; 5A = [[15, 5], [−5, 10]]; 8 − 15 + 7 = 0, 5 − 5 = 0, −5 + 5 = 0, 3 − 10 + 7 = 0 ✓.

Q4. Write A = [[1, 5], [−1, 2]] as symmetric + skew-symmetric.

P = ½(A + A′) = [[1, 2], [2, 2]]; Q = ½(A − A′) = [[0, 3], [−3, 0]].

10 Common mistakes

- Assuming AB = BA, so $(A + B)^2 = A^2 + 2AB + B^2$ (actually $A^2 + AB + BA + B^2$).
- Writing (AB)′ = A′B′ (order reverses).
- Multiplying matrices whose orders don't match.
- Saying a skew-symmetric matrix can have non-zero diagonal elements.

11 Practice questions

1. Order of AB if A is 2×3 and B is 3×4?
2. Find x, y: [[x + y, 2], [5, x − y]] = [[6, 2], [5, 2]].
3. If A is symmetric and B skew-symmetric, is AB − BA symmetric?
4. Number of 2×2 matrices with entries 0 or 1?

ANSWERS

1) 2×4 · 2) x = 4, y = 2 · 3) Yes · 4) 16

12 Quick revision: 5 minutes before the exam

BASICS

- m × n: m rows.
- Equal: same order + elements.

PRODUCT

- (m×n)(n×p) = m×p.
- AB ≠ BA generally.

TRANSPOSE

- $(AB)' = B'A'$.
- $A' = A$ sym; $A' = -A$ skew.

SPLIT

- $A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$.
- $(AB)^{-1} = B^{-1}A^{-1}$.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.