FREE NOTES · NCERT + JEE LEVEL

Inverse Trigonometric Functions

Domain ko seemit kyun karte, chhe inverse functions ke principal value branches, unke graphs, principal values, aur simplify karne wali zaroori properties.

01 Domain ko seemit kyun karte?
02 Principal value branches
03 Graphs
04 Principal values nikalna
05 Properties: self-cancel
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01 Domain ko seemit kyun karte?

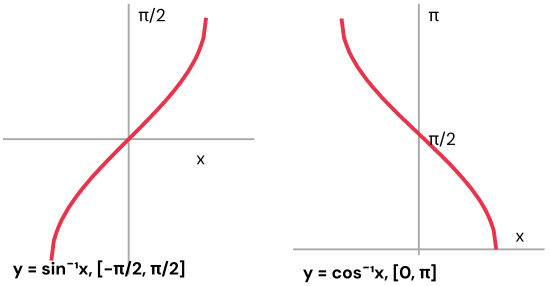
R pe sin x one-one nahi (sin 0 = sin π), isliye wahan inverse nahi. sin ko [−π/2, π/2] tak rakho to ye one-one aur [−1, 1] pe onto ho jaata, aur $\sin^{-1}$ ban jaata. Chuna gaya interval = **principal value branch**.

$\sin^{-1}x \neq (\sin x)^{-1}$. $\sin^{-1}x$ matlab "wo angle jiska sine x hai".

02 Principal value branches

Function	Domain	Range (principal branch)
$\sin^{-1}x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	R	$(-\pi/2, \pi/2)$
$\cot^{-1}x$	R	$(0, \pi)$
$\sec^{-1}x$	$R - (-1, 1)$	$[0, \pi] - \{\pi/2\}$
$\operatorname{cosec}^{-1}x$	$R - (-1, 1)$	$[-\pi/2, \pi/2] - \{0\}$

03 Graphs



$\sin^{-1}x$ aur $\cos^{-1}x$ ke principal branches ($y = x$ line me sin, cos ka mirror)

04 Principal values nikalna

TAREEKA

Principal range me wo angle dhoondo jiski value di gayi ho. $\sin^{-1}(-½) = -\pi/6$ ($7\pi/6$ nahi). $\cos^{-1}(-½) = \pi - \pi/3 = 2\pi/3$. $\tan^{-1}(-\sqrt{3}) = -\pi/3$.

05 Properties: self-cancel

$\sin(\sin^{-1}x) = x$, $x \in [-1, 1]$ (hamesha sahi)

$\sin^{-1}(\sin x) = x$ sirf tab jab $x \in [-\pi/2, \pi/2]$

Example: $\sin^{-1}(\sin 3\pi/5) = \sin^{-1}(\sin 2\pi/5) = 2\pi/5$, kyunki $3\pi/5$ $[-\pi/2, \pi/2]$ se bahar hai. cos (range $[0, \pi]$) aur tan ke liye same idea.

06 Properties: negative, reciprocal, sum

Type	Results
Negative argument	$\sin^{-1}(-x) = -\sin^{-1}x$; $\tan^{-1}(-x) = -\tan^{-1}x$; $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$ $\cos^{-1}(-x) = \pi - \cos^{-1}x$; $\sec^{-1}(-x) = \pi - \sec^{-1}x$; $\cot^{-1}(-x) = \pi - \cot^{-1}x$
Reciprocal	$\sin^{-1}(1/x) = \operatorname{cosec}^{-1}x$; $\cos^{-1}(1/x) = \sec^{-1}x$; $\tan^{-1}(1/x) = \cot^{-1}x$ $(x > 0)$
Complementary	$\sin^{-1}x + \cos^{-1}x = \pi/2$; $\tan^{-1}x + \cot^{-1}x = \pi/2$; $\sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2$
Sum (JEE)	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}[(x + y)/(1 - xy)]$, $xy < 1$ $2\tan^{-1}x = \sin^{-1}(2x/(1+x^2)) = \cos^{-1}((1-x^2)/(1+x^2)) = \tan^{-1}(2x/(1-x^2))$

07 Simplify karne ke tricks

- Andar $\sqrt{(1 - x^2)}$? $x = \sin\theta$ ya $\cos\theta$ rakho.
- $\sqrt{(1 + x^2)}$? $x = \tan\theta$. $\sqrt{(x^2 - 1)}$? $x = \sec\theta$.
- $(1 - \cos x)/(1 + \cos x)$ type: half-angle formulas.
- $(\cos x - \sin x)/(\cos x + \sin x) = \tan(\pi/4 - x)$.

08 JEE Extra + Tricks

JEE LEVEL

- $\sin^{-1}x + \cos^{-1}x = \pi/2$; $\tan^{-1}x + \cot^{-1}x = \pi/2$; $\sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2$.
- $\tan^{-1}x + \tan^{-1}y = \tan^{-1}((x + y)/(1 - xy))$ agar $xy < 1$; $xy > 1$ ($x, y > 0$) pe π jodo.
- $2\tan^{-1}x = \sin^{-1}(2x/(1 + x^2)) = \cos^{-1}((1 - x^2)/(1 + x^2)) = \tan^{-1}(2x/(1 - x^2))$ (ranges ke saath).
- $\sin^{-1}(\sin x) = x$ sirf $x \in [-\pi/2, \pi/2]$ pe; warna principal range me shift karo.
- $\cos^{-1}(-x) = \pi - \cos^{-1}x$; $\sin^{-1}(-x) = -\sin^{-1}x$; $\tan^{-1}(-x) = -\tan^{-1}x$.

09 Solved examples

Q1. $\tan^{-1}(1) + \cos^{-1}(-\frac{1}{2}) + \sin^{-1}(-\frac{1}{2})$ nikalo.

$\pi/4 + 2\pi/3 - \pi/6 = \mathbf{3\pi/4}$.

Q2. $\tan^{-1}(\sqrt{(1 + x^2)} - 1)/x$, $x \neq 0$ simplify karo.

$x = \tan\theta$: $(\sec\theta - 1)/\tan\theta = (1 - \cos\theta)/\sin\theta = \tan(\theta/2) \Rightarrow \mathbf{\frac{1}{2}\tan^{-1}x}$.

Q3. $\cos^{-1}(\cos 7\pi/6)$ nikalo.

$7\pi/6 \notin [0, \pi]$; $\cos 7\pi/6 = \cos 5\pi/6 \Rightarrow \mathbf{5\pi/6}$.

Q4. $\tan^{-1}[(\cos x - \sin x)/(\cos x + \sin x)]$, $-\pi/4 < x < 3\pi/4$ simplify karo.

$= \tan^{-1}\tan(\pi/4 - x) = \mathbf{\pi/4 - x}$.

10 Common mistakes

- Principal range ke bahar answer dena (jaise $\sin^{-1}(-\frac{1}{2}) = 7\pi/6$).
- Har x ke liye $\sin^{-1}(\sin x) = x$ likhna.
- $\sin^{-1}x$ ko $1/\sin x$ samajhna.
- $\cos^{-1}(-x) = -\cos^{-1}x$ likhna ($\pi - \cos^{-1}x$ hai).

11 Practice questions

1. $\text{cosec}^{-1}(2)$ ki principal value.
2. $\sin^{-1}(2x)$ ka domain.
3. $\sin(\pi/3 - \sin^{-1}(-\frac{1}{2}))$ nikalo.
4. $\sin^{-1}x = \pi/5$ ho to $\cos^{-1}x$.

ANSWERS

1) $\pi/6$ · 2) $[-\frac{1}{2}, \frac{1}{2}]$ · 3) 1 · 4) $3\pi/10$

12 Quick revision: exam se 5 minute pehle

RANGES

- $\sin^{-1}$, $\tan^{-1}$: 0 ke aas paas.
- $\cos^{-1}$, $\cot^{-1}$: 0 se π.

NEGATIVE

- sin, tan, cosec: odd.
- cos, sec, cot: π − ....

SUM

- $\sin^{-1} + \cos^{-1} = \pi/2$.
- $\tan^{-1} + \cot^{-1} = \pi/2$.

DHYAN

- $\sin^{-1}(\sin x) = x$ sirf range me.
- Substitution: sin θ, tan θ, sec θ.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.