FREE NOTES · NCERT + JEE LEVEL

Relations & Functions

Relations ke types (reflexive, symmetric, transitive, equivalence), equivalence classes, functions ke types (one-one, onto, bijective), functions ka composition aur invertible functions.

01 Set A pe relations ke types
02 Equivalence relations & classes
03 Functions ke types
04 One-one / onto kaise prove karein
05 Functions ka composition
06 Invertible functions
07 Counting results (n(A) = m, n(B) = n)
08 JEE Extra + Tricks
09 Solved examples
10 Common mistakes
11 Practice questions
12 Quick revision: exam se 5 minute pehle

01 Set A pe relations ke types

Type	Shart
Empty	$R = \varphi$
Universal	$R = A \times A$
Reflexive	Har $a \in A$ ke liye $(a, a) \in R$
Symmetric	$(a, b) \in R \Rightarrow (b, a) \in R$
Transitive	$(a, b), (b, c) \in R \Rightarrow (a, c) \in R$

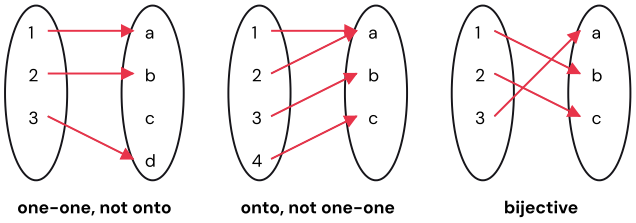
R pe "se chhota": sirf transitive. Ladkon pe "ka bhai": symmetric aur transitive, reflexive nahi.

02 Equivalence relations & classes

Reflexive + symmetric + transitive ⇒ **equivalence relation**. Ye A ko alag alag **equivalence classes** $[a] = \{x \in A : (x, a) \in R\}$ me baant-ta. Har element exactly ek class me.

Example: Z pe $R = \{(a, b) : a - b, 2 \text{ se divisible}\}$ ki classes [0] (even) aur [1] (odd). "Congruent hai", "parallel hai" (line khud ke parallel maan ke) equivalence relations hain.

03 Functions ke types



Teeno types ke arrow diagrams

Type	Matlab
One-one (injective)	Alag inputs ⇒ alag outputs: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
Many-one	One-one nahi
Onto (surjective)	Codomain ke har element ka pre-image: range = codomain
Into	Onto nahi
Bijective	One-one aur onto dono

04 One-one / onto kaise prove karein

TAREEKA

1. **One-one:** $f(x_1) = f(x_2)$ maano, $x_1 = x_2$ dikhao. Ya graph horizontal line test pass kare, ya f strictly increasing/decreasing ho.
2. **Onto:** codomain ka koi bhi y lo, y = f(x) se x nikalo, check karo x domain me hai.

R → R pe $f(x) = x^2$ dono nahi ($f(1) = f(-1)$; -1 ka pre-image nahi). $[0, \infty) \to [0, \infty)$ pe bijective.

05 Functions ka composition

$$(g \circ f)(x) = g(f(x))$$

$f : A \to B$, $g : B \to C \Rightarrow g \circ f : A \to C$. Aam taur pe $g \circ f \neq f \circ g$. Composition associative. f aur g one-one (onto) ⇒ $g \circ f$ one-one (onto).

06 Invertible functions

f : X → Y **invertible** agar g : Y → X ho jisse g ∘ f = I_X aur f ∘ g = I_Y. Tab g = f^{-1}. **f invertible ⇔ f bijective.**

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

f^{-1} nikalna: y = f(x) likho, x ko y me nikalo, wahi $f^{-1}(y)$. Example: f(x) = 4x + 3 ⇒ $f^{-1}(y) = (y - 3)/4$.

07 Counting results (n(A) = m, n(B) = n)

Kitne	Formula
A pe relations	2^{m^2}
A pe reflexive relations	$2^{m^2 - m}$
A pe symmetric relations	$2^{m(m+1)/2}$
Functions A → B	n^m
One-one functions A → B (m ≤ n)	${}^nP_m$
Bijections A → A	m!

08 JEE Extra + Tricks

JEE LEVEL

- n elements pe reflexive relations: $2^{n(n-1)}$; symmetric: $2^{n(n+1)/2}$; kul 2^{n^2}.
- Equivalence relation set ko equivalence classes me baant-ta.
- f invertible ⟺ bijective; $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
- Interval pe strictly monotonic continuous function one-one.
- A se A (n elements) bijections = n!.

09 Solved examples

Q1. {1, 2, 3, 4, 5} pe R = {(a, b) : |a − b| even}. Kya ye equivalence relation hai?

|a − a| = 0 even (reflexive); |a − b| = |b − a| (symmetric); a − b aur b − c even ⇒ a − c even (transitive). Haan. Classes: {1, 3, 5}, {2, 4}.

Q2. Dikhao f : N → N, f(x) = 2x one-one hai par onto nahi.

$2x_1 = 2x_2 \Rightarrow x_1 = x_2$. Par 1 ∈ N ka pre-image nahi (x = ½ ∉ N).

Q3. f(x) = 8x³, g(x) = x^(1/3). g ∘ f aur f ∘ g.

g(f(x)) = 2x; f(g(x)) = 8x.

Q4. Dikhao f : R → R, f(x) = 3x − 5 invertible hai, f^{-1} nikalo.

Linear, slope ≠ 0 ⇒ bijective. y = 3x − 5 ⇒ $f^{-1}(y) = (y + 5)/3$.

10 Common mistakes

- Reflexive sirf kuch elements ke liye check karna (sab ke liye hona chahiye).
- Onto prove karte waqt check na karna ki nikla x domain me hai.
- $(g \circ f)^{-1} = g^{-1} \circ f^{-1}$ likhna (order ulta hota).
- Maan lena ki $g \circ f = f \circ g$.

11 Practice questions

1. {1, 2} pe R = {(1,1), (2,2), (1,2)} symmetric hai?
2. R → R pe f(x) = |x| one-one hai?
3. {a, b} se {1, 2, 3} tak one-one functions?
4. f(x) = (3x + 2)/(5x − 3) ho to dikhao $f \circ f(x) = x$.

ANSWERS

1) Nahi, (2,1) missing · 2) Nahi · 3) 6 · 4) Value daal ke simplify: numerator aur denominator 19x/19 ban jaate

12 Quick revision: exam se 5 minute pehle

RELATIONS

- R, S, T ⇒ equivalence.
- Classes alag alag.

FUNCTIONS

- One-one: $f(x_1)=f(x_2) \Rightarrow x_1=x_2$.
- Onto: range = codomain.

COMPOSITION

- $(g \circ f)(x) = g(f(x))$.
- Commutative nahi.

INVERSE

- Exist ⇔ bijective.
- $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.