FREE NOTES · NCERT + JEE LEVEL

# Relations & Functions

Types of relations (reflexive, symmetric, transitive, equivalence), equivalence classes, types of functions (one-one, onto, bijective), composition of functions and invertible functions.

01 Types of relations on a set A
02 Equivalence relations & classes
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04 How to prove one-one / onto
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## 01 Types of relations on a set A

| Type | Condition |
|---|---|
| Empty | $R = \varphi$ |
| Universal | $R = A \times A$ |
| **Reflexive** | $(a, a) \in R$ for every $a \in A$ |
| **Symmetric** | $(a, b) \in R \Rightarrow (b, a) \in R$ |
| **Transitive** | $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$ |

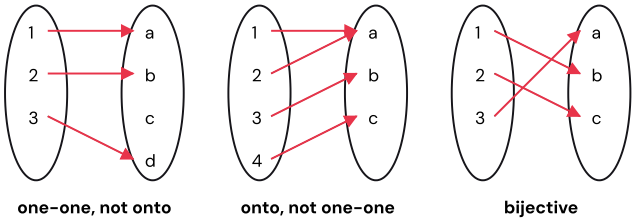
"Is less than" on R: transitive only. "Is a brother of" on boys: symmetric and transitive but not reflexive.

## 02 Equivalence relations & classes

Reflexive + symmetric + transitive $\Rightarrow$ **equivalence relation**. It splits A into disjoint **equivalence classes** $[a] = \{x \in A : (x, a) \in R\}$. Every element is in exactly one class.

Example: $R = \{(a, b) : a - b \text{ divisible by } 2\}$ on Z has classes [0] (even) and [1] (odd). "Is congruent to", "is parallel to" (on lines, allowing a line parallel to itself) are equivalence relations.

## 03 Types of functions




Arrow diagrams for the three types

| Type | Meaning |
|---|---|
| **One-one (injective)** | Different inputs ⇒ different outputs: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ |
| Many-one | Not one-one |
| **Onto (surjective)** | Every element of codomain has a pre-image: range = codomain |
| Into | Not onto |
| **Bijective** | Both one-one and onto |

## 04 How to prove one-one / onto

**METHOD**

1. **One-one:** assume $f(x_1) = f(x_2)$, show $x_1 = x_2$. Or show the graph passes the horizontal line test, or f is strictly increasing/decreasing.
2. **Onto:** take any y in codomain, solve $y = f(x)$ for x, check x lies in the domain.

$f(x) = x^2$ on $R \to R$ is neither ($f(1) = f(-1)$; -1 has no pre-image). On $[0, \infty) \to [0, \infty)$ it is bijective.

## 05 Composition of functions

$$(g \circ f)(x) = g(f(x))$$

$f : A \to B$, $g : B \to C \Rightarrow g \circ f : A \to C$. Usually $g \circ f \neq f \circ g$. Composition is associative. If f and g are one-one (onto), $g \circ f$ is one-one (onto).

## 06 Invertible functions

f : X → Y is **invertible** if there is g : Y → X with $g \circ f = I_X$ and $f \circ g = I_Y$. Then $g = f^{-1}$. **f is invertible ⇔ f is bijective.**

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

To find $f^{-1}$: write y = f(x), solve for x in terms of y, then $f^{-1}(y)$ = that expression. Example: f(x) = 4x + 3 ⇒ $f^{-1}(y) = (y - 3)/4$.

## 07 Counting results (n(A) = m, n(B) = n)

| Number of | Formula |
|---|---|
| Relations on A | $2^{m^2}$ |
| Reflexive relations on A | $2^{m^2 - m}$ |
| Symmetric relations on A | $2^{m(m+1)/2}$ |
| Functions A → B | $n^m$ |
| One-one functions A → B (m ≤ n) | ${}^nP_m$ |
| Bijections A → A | m! |

## 08 JEE Extra + Tricks

**JEE LEVEL**

- Number of reflexive relations on n elements: $2^{n(n-1)}$; symmetric: $2^{n(n+1)/2}$; total relations $2^{n^2}$.
- Equivalence relation partitions the set into equivalence classes.
- f is invertible iff bijective; $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
- Strictly monotonic continuous function on an interval is one-one.
- Number of bijections from A to A (n elements) = n!.

## 09 Solved examples

**Q1.** R = {(a, b) : |a − b| is even} on {1, 2, 3, 4, 5}. Is it an equivalence relation?

|a − a| = 0 even (reflexive); |a − b| = |b − a| (symmetric); a − b and b − c even ⇒ a − c even (transitive). Yes. Classes: {1, 3, 5}, {2, 4}.

**Q2.** Show f : N → N, f(x) = 2x is one-one but not onto.

$2x_1 = 2x_2 \Rightarrow x_1 = x_2$. But 1 ∈ N has no pre-image (x = ½ ∉ N).

**Q3.** f(x) = 8x³, g(x) = x^(1/3). Find g ∘ f and f ∘ g.

g(f(x)) = 2x; f(g(x)) = 8x.

**Q4.** Show f : R → R, f(x) = 3x − 5 is invertible and find $f^{-1}$.

Linear, slope ≠ 0 ⇒ bijective. y = 3x − 5 ⇒ $f^{-1}(y) = (y + 5)/3$.

## 10 Common mistakes

- Checking reflexive for only some elements (must hold for all).
- Proving onto without checking the solved x is in the domain.
- Writing $(g \circ f)^{-1} = g^{-1} \circ f^{-1}$ (order reverses).
- Assuming $g \circ f = f \circ g$.

## 11 Practice questions

1. Is R = {(1,1), (2,2), (1,2)} on {1, 2} symmetric?
2. Is f(x) = |x| on R → R one-one?
3. Number of one-one functions from {a, b} to {1, 2, 3}?
4. If $f(x) = (3x + 2)/(5x - 3)$, show $f \circ f(x) = x$.

**ANSWERS**

1) No, (2,1) missing · 2) No · 3) 6 · 4) Substitute and simplify: numerator and denominator reduce to 19x/19

## 12 Quick revision: 5 minutes before the exam

**RELATIONS**

- R, S, T ⇒ equivalence.
- Classes are disjoint.

**FUNCTIONS**

- One-one: $f(x_1)=f(x_2) \Rightarrow x_1=x_2$.
- Onto: range = codomain.

**COMPOSITION**

- $(g \circ f)(x) = g(f(x))$.
- Not commutative.

**INVERSE**

- Exists ⇔ bijective.
- $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.