FREE NOTES · NCERT + JEE/NEET LEVEL

Waves

Transverse and longitudinal waves, the wave equation, speed on a string and in air, superposition, reflection, standing waves, harmonics of strings and pipes, beats and the Doppler effect.

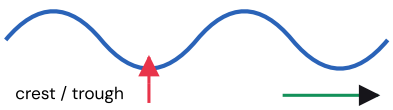
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01 What is a wave

A wave is a **disturbance** that carries energy and momentum from one place to another, while the particles of the medium only oscillate about their positions and don't travel along. Throw a stone in a pond: the ripple reaches the bank, but a floating leaf only bobs up and down.

Type	Needs a medium?	Example
Mechanical	Yes	Sound, waves on a string, water waves
Electromagnetic	No (travels in vacuum)	Light, radio, X-rays
Matter waves	—	Waves associated with electrons, protons

02 Transverse vs longitudinal



Transverse: particles ⊥ to wave direction



Longitudinal: particles ∥ to wave direction

	Transverse	Longitudinal
Particle motion	Perpendicular to wave direction	Along the wave direction
Forms	Crests and troughs	Compressions and rarefactions
Where	Solids, strings, liquid surfaces (not inside fluids)	Solids, liquids and gases
Example	Waves on a string, light	Sound, a push along a spring

03 The wave equation

$$y(x, t) = a \sin(kx - \omega t + \varphi)$$

A wave travelling in +x. With (kx + ωt) it travels in −x.

Quantity	Formula	Meaning
Amplitude a	—	Maximum displacement
Wavelength λ	$k = 2\pi/\lambda$	Distance between two successive crests
Angular wave number k	$2\pi/\lambda$ (rad/m)	—
Frequency ν	$\omega = 2\pi\nu$	Oscillations per second
Wave speed v	$v = \nu\lambda = \omega/k$	How fast the wave moves forward

PHASE DIFFERENCE

Two points Δx apart: $\Delta\varphi = (2\pi/\lambda)\,\Delta x$. Points λ apart are in phase; points λ/2 apart are in opposite phase.

NOTE

Wave speed (v = ω/k) and particle speed (∂y/∂t = −aω cos(…)) are **different**. A particle's maximum speed = aω.

04 Wave speed on a string

$$v = \sqrt{(T/\mu)}$$

T = tension, μ = mass per unit length (kg/m). The tighter the string, the faster the wave. It doesn't depend on frequency (the source decides the frequency).

When a wave enters a new medium its **frequency stays the same**; speed and wavelength change.

05 Speed of sound

GENERAL

Fluid: $v = \sqrt{(B/\rho)}$

Solid rod: $v = \sqrt{(Y/\rho)}$

Speed order: solid > liquid > gas (steel ≈ 5000, water ≈ 1500, air ≈ 340 m/s).

IN A GAS: NEWTON AND LAPLACE

Newton (isothermal): $v = \sqrt{(P/\rho)} \approx 280$ m/s (wrong)

Laplace (adiabatic): $v = \sqrt{(\gamma P/\rho)} = \sqrt{(\gamma RT/M)} \approx 331$ m/s

Compressions in sound are so fast that heat has no time to flow ⇒ adiabatic.

Factor	Effect
Pressure (same T)	No effect (P/ρ constant)
Temperature	$v \propto \sqrt{T}$; rises ≈ 0.61 m/s per °C
Humidity	Speed increases (moist air is lighter)
Wind	Faster in the direction of the wind

06 Principle of superposition

STATEMENT

When two or more waves meet, the net displacement is the **algebraic sum** of the individual displacements: $y = y_1 + y_2 + \ldots$. After meeting, each wave continues unchanged.

SAME FREQUENCY, PHASE DIFFERENCE Φ

$$A = \sqrt{(a_1^2 + a_2^2 + 2a_1a_2 \cos \varphi)}$$

φ = 0, 2π…: **constructive** ($A = a_1 + a_2$). φ = π, 3π…: **destructive** ($A = |a_1 - a_2|$). Intensity ∝ A^2.

07 Reflection of waves

Boundary	What happens
Rigid / fixed end (string tied to a wall)	Wave returns **inverted**: phase change π (crest becomes trough)
Open / free end	Returns with no phase change
From a rarer to a denser medium	Reflected wave has a phase change π

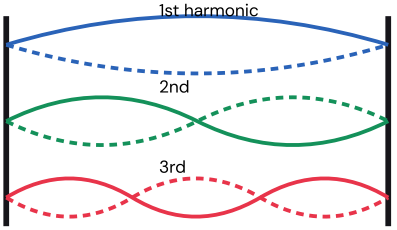
Reflection of sound gives an **echo** (the wall must be at least 17 m away, since the ear separates sounds 0.1 s apart).

08 Standing waves & a vibrating string

Two identical waves travelling in opposite directions form a **standing (stationary) wave**: some points never move (**nodes**), others move the most (**antinodes**). Energy doesn't travel along.

$$y = 2a \sin(kx) \cos(\omega t)$$

Node to node or antinode to antinode = λ/2. Node to antinode = λ/4.



String fixed at both ends

STRING (LENGTH L, BOTH ENDS FIXED)

$$\nu_n = n\, v / (2L),\ n = 1, 2, 3\ldots$$

Fundamental $v_1 = (1/2L)\sqrt{(T/\mu)}$. All harmonics (1 : 2 : 3 …) are present. The sonometer works on this.

09 Organ pipes

	Closed pipe (one end closed)	Open pipe (both ends open)
Ends	Node at the closed end, antinode at the open end	Antinodes at both ends
Frequencies	$\nu = n\, v/(4L),\ n = 1, 3, 5\ldots$	$\nu = n\, v/(2L),\ n = 1, 2, 3\ldots$
Harmonics	Only **odd** (1 : 3 : 5)	All (1 : 2 : 3)
Fundamental	$v/4L$	$v/2L$ (twice the closed pipe's)

END CORRECTION (JEE)

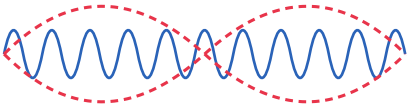
The antinode forms slightly beyond the open end: e ≈ 0.6r. Closed: L + e, open: L + 2e. In a resonance tube: $\lambda/2 = L_2 - L_1$ (the end correction cancels).

10 Beats

Two sounds of slightly different frequencies played together produce a sound that **rises and falls** in loudness (wah-wah).

$$\text{Beat frequency} = |\nu_1 - \nu_2|$$

The ear can distinguish only up to about 10 beats/s.



The rise and fall of amplitude = beats

TUNING FORK TRICK

Load a fork with **wax** ⇒ its frequency falls. **File** it ⇒ its frequency rises. Watch whether beats increase or decrease to find an unknown frequency.

11 Doppler effect (JEE/NEET)

When the source or listener moves, the frequency heard changes. An ambulance sounds higher-pitched approaching and lower moving away.

$$\nu' = \nu\,(v \pm v_o) / (v \mp v_s)$$

v = speed of sound, v_o = observer, v_s = source. When **approaching**, choose signs that raise the frequency: $+v_o$ on top, $-v_s$ below. Reverse when moving apart.

NOTE

If source and observer move with the same velocity (or only the medium moves while both are still), the frequency doesn't change. Moving the source and moving the observer at the same speed don't give the same change.

12 JEE Extra + Tricks

JEE LEVEL

- Wave speed: string $\sqrt{(T/\mu)}$; sound in gas $\sqrt{(\gamma P/\rho)}$ (Laplace correction); $v \propto \sqrt{T}$ (kelvin).
- Open pipe: all harmonics $f_n = nv/2L$; closed pipe: only odd harmonics $f = (2n - 1)v/4L$. End correction $e = 0.6r$ (open end).
- Beats: $f_{beat} = |f_1 - f_2|$; waxing–waning. Filing a tuning fork ⇒ f increases; loading with wax ⇒ f decreases.
- Doppler: $f' = f(v \pm v_o)/(v \mp v_s)$ (upper signs for approach).
- String fixed at both ends: $f_n = nv/2L$, all harmonics; nodes at ends.

13 Solved numericals (v_{sound} = 340 m/s)

Q1. $y = 0.02 \sin(2\pi x - 100\pi t)$ (SI). Amplitude, λ, v and speed?
$a = 0.02$ m. $k = 2\pi \Rightarrow \lambda = 1$ m. $\omega = 100\pi \Rightarrow \nu = 50$ Hz. $v = \nu\lambda = 50$ m/s. **Ans: 2 cm, 1 m, 50 Hz, 50 m/s**

Q2. A string has tension 100 N and μ = 0.01 kg/m. Wave speed?
$v = \sqrt{(100/0.01)} = 100$ m/s. **Ans: 100 m/s**

Q3. A 50 cm string fixed at both ends; wave speed 100 m/s. Fundamental and 3rd harmonic?
$\nu_1 = 100/(2 \times 0.5) = 100$ Hz. $\nu_3 = 300$ Hz. **Ans: 100 Hz, 300 Hz**

Q4. A closed pipe 85 cm long: fundamental and next overtone?
$\nu_1 = 340/(4 \times 0.85) = 100$ Hz. Next: $3\nu_1 = 300$ Hz. **Ans: 100 Hz, 300 Hz**

Q5. Fork A (256 Hz) and B give 4 beats/s. Waxing B makes it 6 beats/s. Frequency of B?
B = 252 or 260. Wax lowers B and beats increase ⇒ 252 → 250 (6 beats) ✓. If 260 → 258, beats would be 2. **Ans: 252 Hz**

Q6. A car with a 500 Hz horn approaches a stationary man at 20 m/s. Frequency he hears?
$\nu' = 500 \times 340/(340 - 20) = 500 \times 340/320 \approx 531$ Hz. **Ans: ≈ 531 Hz**

14 Common mistakes

- Changing the frequency when the medium changes. Frequency stays; v and λ change.
- Thinking the speed of sound depends on pressure.
- Including even harmonics for a closed pipe.
- Confusing wave speed with particle velocity.
- Wrong signs in Doppler: approaching must raise the frequency; check it.
- Reversing the effect of waxing/filing in beat problems.

15 Formula sheet

Wave equation	$y = a\sin(kx - \omega t)$
Relations	$k = 2\pi/\lambda,\ \omega = 2\pi\nu,\ v = \nu\lambda = \omega/k$
String speed	$v = \sqrt{(T/\mu)}$
Sound in a gas	$v = \sqrt{(\gamma P/\rho)} = \sqrt{(\gamma RT/M)}$
Superposition	$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\varphi$
String (both fixed)	$\nu = nv/2L,\ n = 1, 2, 3\ldots$
Closed pipe	$\nu = nv/4L,\ n = 1, 3, 5\ldots$
Open pipe	$\nu = nv/2L,\ n = 1, 2, 3\ldots$
Beats	$\lvert\nu_1 - \nu_2\rvert$
Doppler	$\nu' = \nu(v \pm v_o)/(v \mp v_s)$

16 Practice questions

1. The tension in a string is made 4 times. Fundamental frequency?
2. An open and a closed pipe of the same length. Ratio of fundamentals?
3. Temperature rises from 27 °C to 127 °C. Factor change in the speed of sound?
4. Two waves with amplitudes 3 and 4 have a phase difference of 90°. Resultant amplitude?
5. Phase difference between two points is π/2 and λ = 2 m. Distance between them?

ANSWERS

1) Doubles · 2) open : closed = 2 : 1 · 3) √(400/300) ≈ 1.15 times · 4) 5 · 5) Δx = λΔφ/2π = 0.5 m

17 Quick revision: 5 minutes before the exam

BASICS

- Waves carry energy, not matter.
- Transverse $\perp$, longitudinal $\parallel$.
- $v = \nu\lambda$; new medium $\Rightarrow$ ν unchanged.

SPEED

- String $\sqrt{(T/\mu)}$.
- Gas $\sqrt{(\gamma P/\rho)}$ (Laplace), $\propto \sqrt{T}$.
- Solid > liquid > gas.

STANDING WAVES

- Node–node = $\lambda/2$.
- String/open pipe: $n\nu/2L$ (all).
- Closed pipe: $n\nu/4L$ (odd).
- Fixed end: phase change π.

BEATS & DOPPLER

- Beats = $|\nu_1 - \nu_2|$.
- Wax $\Rightarrow$ $\nu \downarrow$, file $\Rightarrow$ $\nu \uparrow$.
- Approaching $\Rightarrow$ $\nu' \uparrow$.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.