FREE NOTES · NCERT + JEE/NEET LEVEL

Oscillations

Periodic motion, the SHM equation, velocity, acceleration, phase, energy in SHM, the spring-mass system, the simple pendulum and damped/forced oscillations. With graphs and solved numericals.

01 Periodic & oscillatory motion
02 Simple harmonic motion (SHM)
03 Displacement, velocity, acceleration
04 SHM and uniform circular motion
05 Force law
06 Energy in SHM
07 Spring-mass system
08 Simple pendulum
09 Damped & forced oscillations
10 JEE Extra + Tricks
11 Solved numericals (g = 10 m/s², π² ≈ 10)
12 Common mistakes
13 Formula sheet
14 Practice questions
15 Quick revision

01 Periodic & oscillatory motion

- **Periodic:** repeats itself after a fixed time (Earth's rotation, clock hands).
- **Oscillatory:** periodic + to and fro about a **mean position** (swing, pendulum, guitar string).

Every oscillatory motion is periodic, but not every periodic motion is oscillatory (Earth's orbit is not an oscillation).

Time period T: time for one full oscillation (s)

Frequency $\nu = 1/T$ (Hz)

Angular frequency $\omega = 2\pi/T = 2\pi\nu$ (rad/s)

02 Simple harmonic motion (SHM)

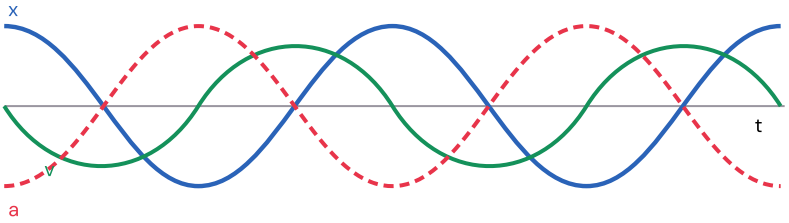
The simplest oscillation, where displacement varies with time like a **sine or cosine**.

$$x(t) = A\cos(\omega t + \varphi)$$

A = **amplitude** (maximum displacement), $(\omega t + \varphi)$ = **phase**, φ = phase constant (where it was at t = 0). Starting from the mean position at t = 0: $x = A \sin \omega t$.

03 Displacement, velocity, acceleration

Quantity	Formula	Mean position (x = 0)	Extreme (x = ±A)
Displacement	$x = A\cos(\omega t + \varphi)$	0	±A
Velocity	$v = -A\omega\sin(\omega t + \varphi) = \omega\sqrt{(A^2 - x^2)}$	Max = Aω	0
Acceleration	$a = -\omega^2 x$	0	Max = Aω²



x (blue), v (green) leads x by π/2, a (red) opposite to x by π

PHASE RELATIONS

Velocity leads displacement by **π/2**. Acceleration is **π** out of phase (exactly opposite). The v–x graph is an **ellipse**; the a–x graph is a straight line with negative slope.

04 SHM and uniform circular motion

A particle moves round a circle of radius A at constant ω. Its **shadow (projection)** on any diameter performs SHM with amplitude A and angular frequency ω. That's why ω in SHM is called "angular frequency".

05 Force law

HOW TO RECOGNISE SHM

F = -k x, a = -ω² x

The restoring force is proportional to displacement and always **toward the mean position**. If a problem gives $a = -(\text{some constant}) \times x$, it is SHM with $\omega = \sqrt{(\text{constant})}$ and $T = 2\pi\sqrt{(m/k)}$.

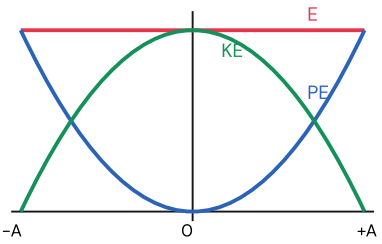
06 Energy in SHM

$KE = \frac{1}{2} k (A^2 - x^2)$

$PE = \frac{1}{2} k x^2$

$$E = \frac{1}{2} k A^2 = \frac{1}{2} m \omega^2 A^2$$

Total energy is constant. All KE at the mean, all PE at the extremes. KE = PE at $x = A/\sqrt{2}$.



KE and PE are parabolas; the total is a straight line

FREQUENCY TRAP

Displacement has frequency ν, but KE and PE oscillate at **2ν** (each peaks twice per oscillation).

07 Spring-mass system

$$T = 2\pi \sqrt{(m/k)}$$

Same T whether horizontal or vertical. Vertically only the mean position shifts down (by mg/k). Independent of g, so it's the same on the Moon.

COMBINATIONS

Series: $1/k = 1/k_1 + 1/k_2$

Parallel: $k = k_1 + k_2$

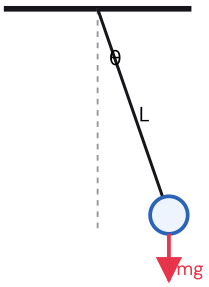
Spring cut into n pieces: $k' = nk$

Two masses joined by a spring (no wall): $T = 2\pi\sqrt{(\mu/k)}$, $\mu = m_1m_2/(m_1 + m_2)$ (reduced mass).

08 Simple pendulum

$$T = 2\pi\sqrt{(L/g)}$$

Only for small angles ($\theta \lesssim 15°$), when $\sin\theta \approx \theta$. **Independent of mass and amplitude**. L = from the point of suspension to the centre of the bob.



Restoring force = mg sin θ

- **Seconds pendulum:** T = 2 s, L ≈ 1 m (99.3 cm).
- Lift accelerating up (a): $g_{eff} = g + a$, T decreases. Down: g − a. Free fall: T = ∞.
- On the Moon g/6 ⇒ T becomes √6 times.
- In summer L increases ⇒ T increases ⇒ the clock runs **slow**.

09 Damped & forced oscillations

Type	What happens
Free oscillation	At its natural frequency, no resistance
Damped	Friction / air removes energy; amplitude falls exponentially: $A\ e^{(-bt/2m)}$
Forced	An external periodic force drives it at its own frequency
Resonance	Driving frequency = natural frequency ⇒ **maximum** amplitude

EXAMPLES OF RESONANCE

Pushing a swing at the right moments; soldiers break step on a bridge (it could collapse); tuning a radio; one tuning fork setting another vibrating.

10 JEE Extra + Tricks

JEE LEVEL

- Springs: series $1/k = 1/k_1 + 1/k_2$; parallel $k = k_1 + k_2$. Vertical spring–mass has the same $T = 2\pi\sqrt{(m/k)}$ as horizontal.
- Pendulum in a lift accelerating up with a: $T = 2\pi\sqrt{(L/(g + a))}$; down: $g - a$; free fall: no oscillation.
- Physical pendulum: $T = 2\pi\sqrt{(I/(mgd))}$; simple pendulum length equal to Earth's radius ⇒ $T \approx 84.6$ min.
- Energy in SHM: $KE = \frac{1}{2}k(A^2 - x^2)$, $PE = \frac{1}{2}kx^2$; KE = PE at $x = A/\sqrt{2}$.
- Phase trick: in SHM, v leads x by $\pi/2$ and a leads x by π ($a = -\omega^2 x$).

11 Solved numericals ($g = 10\ m/s^2$, $\pi^2 \approx 10$)

Q1. $x = 5 \sin(4\pi t + \pi/3)$ cm. Amplitude, frequency, period?
$A = 5$ cm, $\omega = 4\pi \Rightarrow \nu = 2$ Hz, $T = 0.5$ s. **Ans: 5 cm, 2 Hz, 0.5 s**

Q2. In SHM, A = 10 cm, T = 2 s. Maximum velocity and acceleration?
$\omega = \pi$. $v_{max} = A\omega = 0.1\pi \approx 0.31$ m/s. $a_{max} = A\omega^2 = 0.1\pi^2 \approx 1\ m/s^2$. **Ans: 0.31 m/s, ≈ 1 m/s²**

Q3. A 1 kg mass on a spring with k = 100 N/m. Time period?
$T = 2\pi\sqrt{(1/100)} = 0.2\pi \approx 0.63$ s. **Ans: ≈ 0.63 s**

Q4. At what displacement is KE = PE? (A = 4 cm)
$x = A/\sqrt{2} = 2\sqrt{2} \approx 2.83$ cm. **Ans: ≈ 2.83 cm**

Q5. A pendulum's length is made 4 times. T?
$T \propto \sqrt{L} \Rightarrow$ doubles. **Ans: 2T**

Q6. A 1 m pendulum is in a lift accelerating up at 2.5 m/s². T?
$g_{eff} = 12.5$. $T = 2\pi\sqrt{(1/12.5)} = 2\pi \times 0.283 \approx 1.78$ s. **Ans: ≈ 1.78 s**

Q7. A particle has acceleration a = -16x. Time period?
$\omega^2 = 16 \Rightarrow \omega = 4 \Rightarrow T = 2\pi/4 = \pi/2 \approx 1.57$ s. **Ans: π/2 s**

12 Common mistakes

- Thinking a pendulum's T depends on its mass.
- Thinking a spring-mass T depends on g.
- Thinking acceleration is maximum at the mean position (it's 0 there, maximum at the extremes).
- Writing ν for the frequency of KE (it is 2ν).
- Using $T = 2\pi\sqrt{(L/g)}$ for large angles.
- Adding k directly for springs in series (reciprocals add in series).

13 Formula sheet

Displacement	$x = A\cos(\omega t + \varphi)$
Velocity	$v = \omega\sqrt{(A^2 - x^2)}$, $v_{max} = A\omega$
Acceleration	$a = -\omega^2 x$, $a_{max} = A\omega^2$
Force law	$F = -kx$, $\omega = \sqrt{(k/m)}$
Energy	$E = \frac{1}{2}kA^2$, $KE = \frac{1}{2}k(A^2 - x^2)$
Spring	$T = 2\pi\sqrt{(m/k)}$
Pendulum	$T = 2\pi\sqrt{(L/g)}$
Springs	series $1/k = \Sigma 1/k_i$, parallel $k = \Sigma k_i$
Damped amplitude	A e^(−bt/2m)

14 Practice questions

1. The mass on a spring is made 4 times. T?
2. The amplitude of SHM doubles. Total energy?
3. Speed at the mean position is 0.5 m/s and A = 5 cm. ω?
4. A seconds pendulum is taken to the Moon. New T?
5. Two springs (k, k) in series with mass m. T?

ANSWERS

1) 2T · 2) 4 times · 3) $\omega = 0.5/0.05 = 10$ rad/s · 4) $2\sqrt{6} \approx 4.9$ s · 5) $k_{eq} = k/2 \Rightarrow T = 2\pi\sqrt{(2m/k)}$

15 Quick revision: 5 minutes before the exam

SHM

- $a = -\omega^2 x$ (the test).
- $v_{max} = A\omega$ (mean), $a_{max} = A\omega^2$ (extreme).
- v leads x by π/2; a opposite x by π.

ENERGY

- $E = \frac{1}{2}kA^2$, constant.
- KE = PE at $x = A/\sqrt{2}$.
- KE/PE frequency = 2v.

SYSTEMS

- Spring: $2\pi\sqrt{(m/k)}$, independent of g.
- Pendulum: $2\pi\sqrt{(L/g)}$, independent of mass.
- Seconds pendulum L ≈ 1 m.

DAMPED/FORCED

- Damping: amplitude falls exponentially.
- Resonance: driving = natural frequency.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.