FREE NOTES · NCERT + JEE/NEET LEVEL

Kinetic Theory

The gas laws, the ideal gas, assumptions of kinetic theory, the pressure formula, what temperature really means, rms speed, degrees of freedom, equipartition and mean free path.

01 Gas laws
02 Ideal gas equation
03 Assumptions of kinetic theory
04 Pressure of a gas
05 What temperature really means
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01 Gas laws

Law	Constant	Relation
Boyle's law	T, mass	$P \propto 1/V$ ($PV = \text{const}$)
Charles' law	P, mass	$V \propto T$
Gay-Lussac's law	V, mass	$P \propto T$
Avogadro's law	P, T	Equal volumes contain equal numbers of molecules
Dalton's law	V, T	$P_{total} = P_1 + P_2 + \ldots$ (partial pressures)

At STP, 1 mole of gas = 22.4 litres. Avogadro number $N_A = 6.02 \times 10^{23}$.

02 Ideal gas equation

$$PV = \mu R T = N k_B T$$

μ = moles, N = molecules, R = 8.314 J/mol·K, $k_B = R/N_A = 1.38 \times 10^{-23}$ J/K. In terms of density: $P = \rho RT/M$ (M = molar mass).

WHEN A REAL GAS BEHAVES IDEALLY

At **low pressure and high temperature**. Then molecules are far apart and their mutual attraction is negligible.

03 Assumptions of kinetic theory

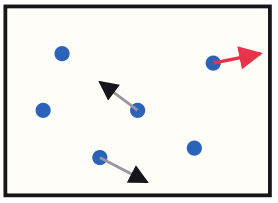
1. A gas consists of a huge number of tiny molecules whose own volume is negligible.
2. Molecules move randomly, in all directions, at various speeds.
3. Collisions (with each other and the walls) are **perfectly elastic**.
4. No forces act between molecules except during collisions.
5. Collision time is very short compared with the time between collisions.
6. Newton's laws apply.

04 Pressure of a gas

Molecules bounce off the walls, each collision giving momentum $2mv_x$ to the wall. The average force of millions of collisions = pressure.

$$P = (1/3)\, n\, m\, \bar{v}^2 = (1/3)\, \rho\, {v_{rms}}^2$$

n = molecules per unit volume. From this: $P = (2/3)\, E$, E = KE per unit volume.



Molecules hitting the walls = pressure

05 What temperature really means

KINETIC INTERPRETATION

Average KE per molecule = $(3/2)\, k_B\, T$

Temperature measures the average translational KE of the molecules. All gases at the same T have the same average KE (independent of mass). KE of 1 mole = (3/2) RT.

At absolute zero (0 K), according to classical theory, molecular motion stops.

06 Molecular speeds

Speed	Formula	Ratio
RMS speed	$v_{rms} = \sqrt{(3RT/M)} = \sqrt{(3k_BT/m)}$	√3 ≈ 1.73
Average speed	$v_{avg} = \sqrt{(8RT/\pi M)}$	√(8/π) ≈ 1.60
Most probable speed	$v_{mp} = \sqrt{(2RT/M)}$	√2 ≈ 1.41

ORDER

$v_{rms} > v_{avg} > v_{mp}$. All $\propto \sqrt{(T/M)}$.

SHORTCUTS

- Temperature × 4 ⇒ v_{rms} × 2.
- At the same T a lighter gas is faster: H_2's v_{rms} is 4 times O_2's (√(32/2)).
- Increasing pressure (same T) doesn't change v_{rms}.
- Air molecules at room temperature move at ≈ 500 m/s.

07 Degrees of freedom (f)

The number of independent ways a molecule can store energy.

Molecule	Translational	Rotational	Total f (ordinary temperatures)
Monoatomic (He, Ne)	3	0	3
Diatomic (O_2, N_2)	3	2	5 (7 at high T with vibration)
Polyatomic non-linear (H_2O, CH_4)	3	3	6

08 Law of equipartition of energy

STATEMENT

In thermal equilibrium each degree of freedom gets energy **½ k_BT** (a vibrational mode gets k_BT, since it has both KE and PE).

Energy of one molecule = $(f/2)\, k_BT$

Internal energy of 1 mole $U = (f/2)\, RT$

09 Specific heats from kinetic theory

$C_v = (f/2)\,R$, $C_p = (f/2 + 1)\,R$

$$\gamma = 1 + 2/f$$

Gas	f	C_v	C_p	γ
Monoatomic	3	3R/2	5R/2	1.67
Diatomic (rigid)	5	5R/2	7R/2	1.4
Polyatomic (non-linear)	6	3R	4R	1.33

MIXTURE OF GASES

$C_{v,mix} = (n_1 C_{v1} + n_2 C_{v2})/(n_1 + n_2)$. Find γ_{mix} from this, not by averaging γ directly.

Solids: each atom vibrates in 3 directions, each vibration with energy $k_B T \Rightarrow C = 3R$ (Dulong-Petit).
Water: C ≈ 9R (very high).

10 Mean free path

The average distance a molecule travels in a straight line between two successive collisions.

$$\lambda = 1 / (\sqrt{2}\,\pi\, n\, d^2) = k_B T / (\sqrt{2}\,\pi\, d^2\, P)$$

n = number density, d = molecular diameter. Higher density or pressure ⇒ smaller λ. In air at STP λ ≈ 0.1 μm (~500 times a molecule's size).

That's why a perfume's smell spreads slowly, not instantly: the molecules are fast but collide constantly.

11 JEE Extra + Tricks

JEE LEVEL

- Speeds: $v_{rms} = \sqrt{(3RT/M)}$, $v_{avg} = \sqrt{(8RT/\pi M)}$, $v_{mp} = \sqrt{(2RT/M)}$; ratio ≈ 1.73 : 1.60 : 1.41 (rms > avg > mp).
- Degrees of freedom f: monatomic 3, diatomic 5 (7 at high T with vibration); $\gamma = 1 + 2/f \Rightarrow$ 5/3, 7/5, 4/3.
- Mixture of gases: $C_{v,mix} = (n_1C_{v1} + n_2C_{v2})/(n_1 + n_2)$.
- Mean free path $\lambda = 1/(\sqrt{2}\,\pi\, d^2\, n) = kT/(\sqrt{2}\,\pi\, d^2\, P)$.
- Pressure $P = \frac{1}{3}\rho v_{rms}^2$; average KE per molecule = (3/2)kT, independent of nature of gas.

12 Solved numericals (R = 8.3 J/mol·K)

Q1. v_{rms} of O_2 (M = 0.032 kg/mol) at 300 K?
$v = \sqrt{(3 \times 8.3 \times 300/0.032)} = \sqrt{233437} \approx 483$ m/s. **Ans: ≈ 483 m/s**

Q2. At what temperature does O_2 have the same v_{rms} as H_2 at 27 °C?
T/M equal ⇒ $T_{O_2} = 300 \times 32/2 = 4800$ K. **Ans: 4800 K**

Q3. Average translational KE of a molecule at 27 °C? ($k_B = 1.38 \times 10^{-23}$)
$KE = 1.5 \times 1.38 \times 10^{-23} \times 300 \approx 6.2 \times 10^{-21}$ J. **Ans: ≈ 6.2×10^{-21} J**

Q4. A mixture of 2 mol He and 1 mol O_2. γ_{mix}?
$C_v = (2 \times 1.5R + 1 \times 2.5R)/3 = 5.5R/3$. $C_p = C_v + R = 8.5R/3$. $\gamma = 8.5/5.5 \approx 1.55$. **Ans: ≈ 1.55**

Q5. A gas in a cylinder is at 2 atm and 300 K. It is heated to 600 K at constant volume. Pressure?
$P \propto T \Rightarrow 4$ atm. **Ans: 4 atm**

13 Common mistakes

- Putting M in g/mol in v_{rms}. Use kg/mol (O_2 = 0.032).
- Using °C. Use kelvin.
- Averaging γ directly for a mixture.
- Thinking average KE depends on the gas's mass (only on T).
- Counting vibrational degrees for a diatomic gas at ordinary temperatures.

14 Formula sheet

Ideal gas	$PV = \mu RT = Nk_BT$
Pressure	$P = \frac{1}{3}\rho v_{rms}^2 = \frac{2}{3}E$
Average KE	$(3/2)k_BT$ per molecule
Speeds	$\sqrt{(3RT/M)}$, $\sqrt{(8RT/\pi M)}$, $\sqrt{(2RT/M)}$
Equipartition	$\frac{1}{2}k_BT$ per degree of freedom
Internal energy	$U = (f/2)\mu RT$
Specific heats	$C_v = fR/2$, $\gamma = 1 + 2/f$
Mean free path	$\lambda = 1/(\sqrt{2}\,\pi nd^2)$

15 Practice questions

1. Temperature rises from 27 °C to 927 °C. By what factor does v_{rms} change?
2. Internal energy of 1 mole of diatomic gas at 300 K?
3. A gas has γ = 1.4. Degrees of freedom?
4. Pressure doubles (same T). Mean free path?

ANSWERS

1) T 300 → 1200 K ⇒ doubles · 2) (5/2) × 8.3 × 300 ≈ 6.2 kJ · 3) 1.4 = 1 + 2/f ⇒ f = 5 · 4) Halves

16 Quick revision: 5 minutes before the exam

GAS LAWS

- $PV = \mu RT$; real gases are ideal at low P, high T.
- Boyle, Charles, Gay-Lussac, Dalton.
- $k_B = R/N_A$.

KINETIC THEORY

- $P = \frac{1}{3}\rho v^2_{rms}$.
- $KE = (3/2)k_BT$, independent of mass.
- $v_{rms} > v_{avg} > v_{mp}$, $\propto \sqrt{(T/M)}$.

EQUIPARTITION

- f: mono 3, di 5, poly 6.
- $\frac{1}{2}k_BT$ per degree.
- $\gamma = 1 + 2/f$.

OTHERS

- Solid $C = 3R$.
- $\lambda = 1/(\sqrt{2}\,\pi n d^2)$.
- Mixture: weighted average of C_v.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.