FREE NOTES · NCERT + JEE/NEET LEVEL

Mechanical Properties of Solids

Elasticity, stress, strain, Hooke's law, the stress-strain curve, Young's, shear and bulk modulus, Poisson's ratio and stored elastic energy. With everyday examples and solved numericals.

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01 Elasticity & plasticity

Elasticity: a body regains its original shape and size when the force is removed (rubber band, spring).
Plasticity: it does not, and stays permanently deformed (wet clay, putty, chewing gum).

WHAT HAPPENS INSIDE

The atoms of a solid are bound to each other like springs. Stretch it and the spacing increases, creating a **restoring force** that pulls back.

INTERESTING FACT

Steel is more elastic than rubber. Being elastic doesn't mean "stretching more" but "resisting deformation more". For the same stress steel has much less strain, so its Young's modulus is larger. Quartz is treated as nearly perfectly elastic.

02 Stress

The restoring force per unit area developed inside a deformed body.

$$\text{Stress} = F / A$$

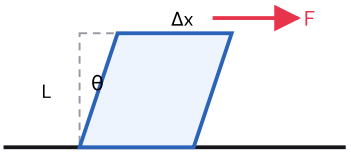
Unit N/m² = Pa. Dimensions $[ML^{-1}T^{-2}]$ (like pressure).

Type	How the force acts	What changes
Longitudinal (tensile / compressive)	Perpendicular to the surface, pulling or pushing	Length
Shearing (tangential)	Parallel to the surface	Shape (a cube becomes slanted)
Hydraulic (volume)	Equal pressure from all sides (body under water)	Volume

03 Strain

The fractional deformation (change / original). It is a ratio, so it has **no unit or dimension**.

$$\text{Longitudinal strain} = \Delta L / L$$
$$\text{Shearing strain} = \Delta x / L = \tan\theta \approx \theta$$
$$\text{Volume strain} = \Delta V / V$$



Shear: the top face slides by Δx

04 Hooke's law

STATEMENT

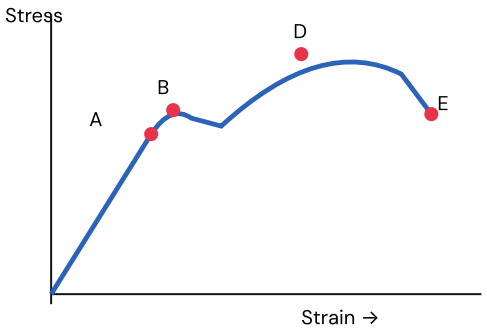
For small deformations (within the elastic limit), **stress ∝ strain**.

Stress = k × Strain

k = **modulus of elasticity**. Its unit and dimensions are those of stress (Pa), since strain has no unit.

A modulus depends on the **material**, not on the size or shape of the sample. Moduli usually decrease as temperature rises.

05 Stress-strain curve



Typical curve for a metal (like steel)

Point	Meaning
O–A	Straight line: Hooke's law (proportional limit)
A–B	Elastic but not proportional; B = **elastic limit / yield point**
B–D	Plastic region: permanent set
D	**Ultimate tensile strength** (maximum stress)
E	**Fracture point** (breaks)

Material	How the curve looks
Ductile (copper, aluminium)	Large gap between D and E: can be drawn into thin wires
Brittle (glass, cast iron)	D and E very close: breaks soon after
Elastomer (rubber, aorta)	Doesn't obey Hooke's law, but returns even after very large stretching

06 Young's modulus (Y)

$$Y = (F/A) / (\Delta L/L) = F\,L / (A\,\Delta L)$$

For a wire of radius r: $\Delta L = FL/(\pi r^2 Y)$. Defined only for **solids**. Steel ≈ 2 × 10^{11} Pa, aluminium ≈ 7 × 10^{10} Pa.

SHORTCUT RATIOS

- Same material, same force: $\Delta L \propto L/r^2$.
- Stretch a wire to double length keeping volume constant: area halves ⇒ ΔL becomes 4 times.
- A rope hanging under its own weight: $\Delta L = \rho g L^2/(2Y)$ (take half the weight).

Searle's apparatus measures Y in the lab: a reference wire and a test wire, with a spirit level and micrometer to read the small extension.

07 Shear & bulk modulus

SHEAR MODULUS (RIGIDITY, G OR H)

$$G = (F/A) / \theta$$

Resistance to change of shape. Usually G ≈ Y/3. Liquids and gases have G = 0 (no fixed shape).

BULK MODULUS (B)

$$B = -p / (\Delta V/V)$$

Minus because volume decreases when pressure increases. Compressibility k = 1/B. B order: solid > liquid > gas. Gases compress most easily.

State	Y	G	B
Solid	Yes	Yes	Yes
Liquid	No	No	Yes
Gas	No	No	Yes (isothermal B = p, adiabatic B = γp)

08 Poisson's ratio (σ)

Stretch rubber and it also gets thinner. The ratio of lateral strain to longitudinal strain:

$$\sigma = -(\Delta d/d) / (\Delta L/L)$$

Unitless. In theory -1 to 0.5, in practice 0 to 0.5. σ = 0.5 means volume doesn't change (incompressible, e.g. rubber ≈ 0.5). For metals ≈ 0.3.

RELATIONS BETWEEN MODULI (JEE)

$$Y = 2G(1+\sigma) = 3B(1-2\sigma)$$

09 Elastic potential energy

The work done in stretching a wire is stored inside as **elastic PE**.

$$U = \tfrac{1}{2} \times F \times \Delta L = \tfrac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

Energy density (per unit volume) $u = \tfrac{1}{2} \times \text{stress} \times \text{strain} = \text{stress}^2/(2Y) = \tfrac{1}{2}\, Y\, \text{strain}^2$

Area under the stress-strain graph = energy density.

THERMAL STRESS

Heat a rod with both ends fixed by ΔT; it can't expand, so stress develops: stress $= Y\,\alpha\,\Delta T$, force = $YA\alpha\Delta T$. That's why gaps are left between railway rails.

10 Applications (NCERT)

- **Crane rope:** keep the load's stress below the yield strength, with a safety factor. Ropes are made of many thin wires twisted together instead of one thick wire (flexible and strong).
- **Beams in bridges and buildings:** sag under a central load W: $\delta = WL^3/(4bd^3Y)$. Increasing the depth d helps most (d^3), so beams have an **I-shaped** cross-section: less material, less sag, less weight.
- **Maximum height of a mountain:** the stress hρg on the rock at the base can't exceed its elastic limit, so mountains on Earth can't be much taller than about 10 km.
- A **hollow shaft** is stronger (resists twisting better) than a solid shaft made of the same amount of material.

11 JEE Extra + Tricks

JEE LEVEL

- Relation of elastic constants: $Y = 3K(1 - 2\sigma) = 2\eta(1 + \sigma)$; Poisson's ratio σ between -1 and 0.5 (practically 0 to 0.5).
- Elastic PE per unit volume = $\frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} Y (\text{strain})^2$.
- Thermal stress in a rod fixed at ends: $Y \alpha \Delta T$ (independent of length and area).
- Elongation of a hanging rod under its own weight: $\Delta L = MgL/(2AY)$ (half of the load at end).
- Steel is more elastic than rubber (higher Y). Compressibility = $1/K$.

12 Solved numericals (g = 10 m/s²)

Q1. A 2 m steel wire of area 1 mm² carries 100 N. Y = 2 × 10^{11} Pa. Extension?
$\Delta L = FL/(AY) = 100 \times 2/(10^{-6} \times 2 \times 10^{11}) = 200/(2 \times 10^{5}) = 10^{-3}$ m = 1 mm. **Ans: 1 mm**

Q2. Two wires of the same material: lengths 1 : 2, radii 2 : 1. Ratio of extensions for the same load?
$\Delta L \propto L/r^2 \Rightarrow (1/4) : (2/1) = 1 : 8$. **Ans: 1 : 8**

Q3. What pressure compresses water by 1%? (B = 2.2 × 10^{9} Pa)
$p = B \times \Delta V/V = 2.2 \times 10^{9} \times 0.01 = 2.2 \times 10^{7}$ Pa (≈ 220 atm). **Ans: 2.2 × 10^{7} Pa**

Q4. A 10 cm cube has a 10^{4} N force parallel to its top face. G = 2.5 × 10^{10} Pa. How far does the top face shift?
Stress = $10^{4}/0.01 = 10^{6}$ Pa. $\theta = 10^{6}/(2.5 \times 10^{10}) = 4 \times 10^{-5}$. $\Delta x = \theta L = 4 \times 10^{-6}$ m. **Ans: 4 µm**

Q5. Energy stored in the wire of Q1?
$U = \frac{1}{2} F \Delta L = \frac{1}{2} \times 100 \times 10^{-3} = 0.05$ J. **Ans: 0.05 J**

Q6. A steel rod (Y = 2 × 10^{11}, α = 1.2 × 10^{-5} /°C) is fixed at both ends and heated by 50 °C. Thermal stress?
Stress = $Y\alpha\Delta T = 2 \times 10^{11} \times 1.2 \times 10^{-5} \times 50 = 1.2 \times 10^{8}$ Pa. **Ans: 1.2 × 10^{8} Pa**

13 Common mistakes

- Thinking rubber is more elastic than steel.
- Writing a unit for strain. It is unitless.
- Trying to find Young's modulus of a liquid. Liquids have only a bulk modulus.
- Converting mm² to m² wrongly (1 mm² = 10^{-6} m²).
- Forgetting the minus sign in the bulk modulus formula.
- Thinking a modulus depends on the wire's length or area. It depends only on the material.

14 Formula sheet

Stress, strain	F/A, $\Delta L/L$
Young's modulus	$Y = FL/(A\Delta L)$
Shear modulus	$G = (F/A)/\theta$
Bulk modulus	$B = -p/(\Delta V/V)$, $k = 1/B$
Poisson's ratio	$\sigma = -(\Delta d/d)/(\Delta L/L)$
Relations	$Y = 2G(1+\sigma) = 3B(1-2\sigma)$
Elastic energy	$U = \frac{1}{2}F\Delta L = \frac{1}{2}$ stress $\times$ strain $\times$ V
Energy density	$u = \text{stress}^2/2Y$
Thermal stress	$Y\alpha\Delta T$
Beam sag	$\delta = WL^3/(4bd^3Y)$

15 Practice questions

1. Both the length and radius of a wire are doubled (same load). Extension?
2. If stress is doubled (within the elastic limit), what happens to energy density?
3. Young's modulus of a perfectly rigid body?
4. What does a Poisson's ratio of 0.5 for rubber mean?
5. A 4 m wire of area 2 mm² stretches 2 mm under 200 N. Find Y.

ANSWERS

1) $\Delta L \propto L/r^2 \Rightarrow 2/4$ = half · 2) 4 times · 3) Infinite · 4) Volume doesn't change on stretching · 5) $Y = 200 \times 4/(2 \times 10^{-6} \times 2 \times 10^{-3}) = 2 \times 10^{11}$ Pa

16 Quick revision: 5 minutes before the exam

BASICS

- Stress = F/A (Pa); strain is unitless.
- Hooke: stress ∝ strain.
- Steel > rubber (elasticity).

MODULI

- $Y = FL/A\Delta L$; $\Delta L \propto L/r^2$.
- $G \approx Y/3$; $G = 0$ for liquids.
- $B = -p/(\Delta V/V)$; for a gas $B = p$ (isothermal).
- $Y = 2G(1 + \sigma) = 3B(1 - 2\sigma)$.

CURVE

- Proportional → elastic/yield → ultimate → fracture.
- Ductile: D–E far apart; brittle: close.
- Elastomers don't obey Hooke's law.

ENERGY & USES

- $U = \frac{1}{2}F\Delta L$; $u = \text{stress}^2/2Y$.
- Thermal stress $Y\alpha\Delta T$.
- I-beam: sag ∝ $1/d^3$.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.