FREE NOTES · NCERT + JEE/NEET LEVEL

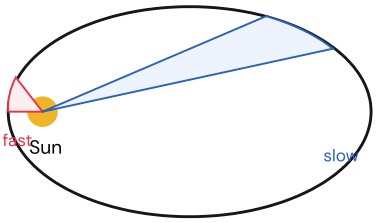
Gravitation

Kepler's laws, Newton's law of gravitation, how g changes, gravitational potential, escape speed, satellites and weightlessness. Explained simply, with graphs and solved numericals.

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01 Kepler's laws

Law	What it says
1. Law of orbits	Every planet moves in an **ellipse** around the Sun, with the Sun at one focus.
2. Law of areas	The line from Sun to planet sweeps **equal areas in equal times**. So a planet is faster near the Sun (perihelion) and slower far away (aphelion).
3. Law of periods	$T^2 \propto a^3$ (a = semi-major axis)



Both shaded areas are swept in equal times

WHAT THE 2ND LAW MEANS

Areal velocity $dA/dt = L/(2m)$ is constant, i.e. **angular momentum is conserved** (gravity is a central force, so torque is zero). At perihelion and aphelion: $v_1 r_1 = v_2 r_2$.

02 Universal law of gravitation

$$F = G\, m_1\, m_2 \,/\, r^2$$

$G = 6.67 \times 10^{-11}$ N·m²/kg² (universal constant, measured by Cavendish). The force is always attractive, along the line joining the masses.

- A very weak force, but with infinite range.
- Independent of the medium in between.
- An action-reaction pair: the Earth pulls an apple as much as the apple pulls the Earth.
- For several masses, find each force separately and take the **vector sum** (superposition).

SHELL THEOREM

At a point **outside** a uniform hollow shell: treat all its mass as at the centre. Anywhere **inside** the shell: force is zero.

03 G vs g

	G	g
What it is	Universal gravitational constant	Acceleration due to gravity
Value	6.67×10^{-11} N·m²/kg², same everywhere	≈ 9.8 m/s², varies with place
Type	Scalar constant	Vector

RELATION

$$g = G\,M\,/\,R^2$$

M, R = Earth's mass and radius. With density ρ: $g = (4/3)\pi G\rho R$. It does not depend on the mass of the falling body.

04 g with height

AT HEIGHT H

$$g_h = g\,R^2\,/\,(R+h)^2$$

Small height ($h \ll R$): $g_h \approx g(1 - 2h/R)$.

AT DEPTH D

$$g_d = g\,(1 - d/R)$$

At the centre (d = R): g = 0.

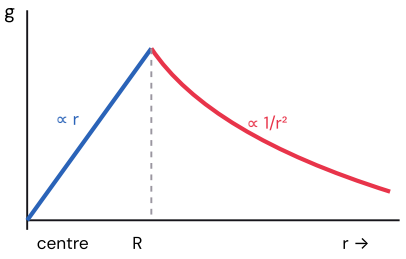
05 g with depth, and the full graph

Going down, only the mass of the inner sphere matters (the outer shell gives zero force). So g decreases **linearly** and becomes 0 at the centre.

g is **maximum at the surface**. It decreases whether you go up or down.

COMPARISON

For a small distance x: decrease at height x ≈ 2x/R, at depth x = x/R. So g falls **faster with height**.



Inside g ∝ r, outside g ∝ 1/r²

06 Effect of rotation & latitude

$$g' = g - \omega^2 R \cos^2\lambda$$

λ = latitude, ω = Earth's angular velocity. At the equator (λ = 0) g is least, g - ω²R. At the poles (λ = 90°) rotation has no effect, so g is greatest.

- The Earth is slightly flattened at the poles ($R_{pole} < R_{equator}$), another reason g is larger there.
- If the Earth spun at ω = √(g/R) ≈ 1/800 rad/s (about 17 times faster), objects at the equator would become weightless.
- If the Earth stopped rotating, g at the poles wouldn't change; at the equator it would increase.

07 Gravitational field & potential

FIELD (INTENSITY)

$$E = F/m = GM/r^2$$

Force on a unit mass. A vector, pointing toward the mass. At the surface E = g.

POTENTIAL

$$V = -GM/r$$

Work to bring a unit mass from infinity to r. A scalar, always negative (0 at infinity). Relation: $E = -dV/dr$.

Body	Inside (r < R)	Outside (r ≥ R)
Hollow shell: field	0	GM/r^2
Hollow shell: potential	$-GM/R$ (constant)	$-GM/r$
Solid sphere: field	GMr/R^3	GM/r^2
Solid sphere: potential	$-GM(3R^2 - r^2)/(2R^3)$	$-GM/r$

At the centre of a solid sphere the potential is $-3GM/(2R)$, 1.5 times the surface value (the lowest).

08 Gravitational potential energy

$$U = -G\,M\,m\,/\,r$$

U = 0 at infinity, so it is negative closer in. Negative means the system is **bound** (energy must be supplied to separate it).

WHEN DOES MGH WORK?

Change in PE when raised h from the surface: $\Delta U = GMm\,h/(R(R+h)) = mgh/(1+h/R)$. Only when **h ≪ R** is it ≈ mgh. For large heights mgh gives a wrong answer.

For more than two particles, find U for each pair and add. Three equal masses m at the corners of a triangle of side a: $U = -3Gm^2/a$.

09 Escape speed

The minimum speed from the surface so that a body never returns (goes to infinity).

$$v_e = \sqrt{(2GM/R)} = \sqrt{(2gR)}$$

Earth: ≈ 11.2 km/s. Moon: ≈ 2.3 km/s (so the Moon has no atmosphere; gas molecules escaped). Independent of the body's mass and the direction of projection.

DERIVATION (ONE LINE)

$\frac{1}{2}mv_e^2 - GMm/R = 0$ (both KE and PE are 0 at infinity) ⇒ $v_e = \sqrt{(2GM/R)}$.

10 Satellites

Gravity provides the centripetal force: $mv^2/r = GMm/r^2$, with r = R + h.

ORBITAL SPEED & PERIOD

$$v_0 = \sqrt{(GM/r)}$$

Near the surface: $v_0 = \sqrt{(gR)} \approx 7.9\ km/s$

$$T = 2\pi\sqrt{(r^3/GM)}$$

Independent of the satellite's mass. Higher satellites are slower and take longer. $v_e = \sqrt{2} \times v_o$.

ENERGIES

$KE = GMm/(2r)$

$PE = -GMm/r$

$Total\ E = -GMm/(2r)$

$KE : PE : E = 1 : -2 : -1$

Binding energy = GMm/(2r). Energy to move from r to 2r = GMm/(4r).

	Geostationary	Polar
Height	≈ 36,000 km (r ≈ 42,250 km)	≈ 500–800 km
Period	24 h (moves with the Earth)	≈ 100 min
Orbit	Above the equator, west → east	Over the poles (north–south)
Use	TV, communication	Weather, remote sensing, surveillance

11 Weightlessness

In a satellite, the astronaut and the satellite are **falling with the same acceleration** (free fall). So the astronaut doesn't press on the floor, the normal force is 0, and no weight is felt. Gravity is **not** zero.

NOTE

In a satellite a pendulum won't swing (T = ∞) and water can't be poured into a glass, but mass can still be measured with an inertial balance instead of a spring balance.

12 JEE Extra + Tricks

JEE LEVEL

- g at height h ($h \ll R$): $g(1 - 2h/R)$; at depth d: $g(1 - d/R)$ (zero at centre); due to rotation at latitude λ: $g - \omega^2 R \cos^2\lambda$.
- Escape speed $v_e = \sqrt{(2GM/R)} \approx 11.2$ km/s = $\sqrt{2}$ × orbital speed near surface.
- Satellite: KE = GMm/2r, PE = −GMm/r = −2KE, total E = −KE (bound). Time period $T = 2\pi\sqrt{(r^3/GM)}$.
- Geostationary: ~36,000 km above surface, T = 24 h, equatorial orbit, west to east.
- Kepler: $T^2 \propto r^3$; areal velocity constant (angular momentum conserved).
- Shell theorem: inside a hollow shell field = 0, potential constant.

13 Solved numericals ($g = 9.8\ m/s^2$, $R = 6400$ km)

Q1. Two 100 kg spheres are 1 m apart. Gravitational force?
$F = 6.67 \times 10^{-11} \times 10^4/1 = 6.67 \times 10^{-7}$ N (very small!). **Ans: 6.67×10^{-7} N**

Q2. At what height is g one quarter of its surface value?
$R^2/(R + h)^2 = ¼ \Rightarrow R + h = 2R \Rightarrow h = R = 6400$ km. **Ans: 6400 km**

Q3. At what depth is g 75% of its surface value?
$1 - d/R = 0.75 \Rightarrow d = R/4 = 1600$ km. **Ans: 1600 km**

Q4. A planet has twice the Earth's mass and twice its radius. g there?
$g \propto M/R^2 \Rightarrow 2/4 = ½ \Rightarrow 4.9\ m/s^2$. **Ans: g/2**

Q5. Escape speed of the same planet compared with Earth (11.2 km/s)?
$v_e \propto \sqrt{(M/R)} \Rightarrow \sqrt{(2/2)} = 1 \Rightarrow 11.2$ km/s. **Ans: the same, 11.2 km/s**

Q6. A satellite orbits at height R above the surface. Orbital speed? ($\sqrt{(gR)} = 7.9$ km/s)
$r = 2R \Rightarrow v = \sqrt{(GM/2R)} = \sqrt{(gR/2)} = 7.9/\sqrt{2} \approx 5.6$ km/s. **Ans: ≈ 5.6 km/s**

Q7. A satellite's period is T. If the orbital radius is made 4 times, the period?
$T^2 \propto r^3 \Rightarrow T' = T \times 4\text{^}(3/2) = 8T$. **Ans: 8T**

Q8. Energy needed to put a 1000 kg satellite from the surface into an orbit of radius 2R ($GMm/R = mgR$)?
$E_{orbit} - E_{surface} = -GMm/(4R) - (-GMm/R) = 3GMm/(4R) = ¾\ mgR = 0.75 \times 1000 \times 9.8 \times 6.4 \times 10^6 \approx 4.7 \times 10^{10}$ J. **Ans: ≈ 4.7×10^{10} J**

14 Common mistakes

- Thinking weightlessness in a satellite means "zero gravity".
- Using mgh for the change in PE at large heights.
- Thinking g is infinite at the Earth's centre. It is 0.
- Thinking escape speed depends on direction.
- Thinking a satellite's period or speed depends on its mass.
- Confusing height h with r (from the centre): $r = R + h$.

15 Formula sheet

Newton's law	$F = Gm_1m_2/r^2$
g at the surface	$g = GM/R^2 = (4/3)\pi G\rho R$
Height / depth	$g(1 - 2h/R)$ / $g(1 - d/R)$
Latitude	$g' = g - \omega^2 R\cos^2\lambda$
Field, potential	$E = GM/r^2$, $V = -GM/r$
Potential energy	$U = -GMm/r$
Escape speed	$v_e = \sqrt{(2gR)} \approx 11.2$ km/s
Orbital speed	$v_0 = \sqrt{(GM/r)}$, near surface $\sqrt{(gR)}$
Period	$T = 2\pi\sqrt{(r^3/GM)}$, $T^2 \propto r^3$
Satellite energy	$E = -GMm/2r$, $KE = -E$
Kepler 2nd	$dA/dt = L/2m = \text{constant}$

16 Practice questions

1. If the distance between two masses is halved, what happens to the force?
2. A planet has the same density as Earth but twice the radius. Its g?
3. A planet's orbital radius is 4 times the Earth's. How long is its year?
4. A satellite's KE is 5×10^9 J. Find its total energy and PE.
5. If the Earth shrank to half its radius (same mass), escape speed?

ANSWERS

1) 4 times · 2) g ∝ ρR ⇒ 2g · 3) 4^(3/2) = 8 years · 4) E = -5 × 10^9 J, PE = -10^{10} J · 5) √2 × 11.2 ≈ 15.8 km/s

17 Quick revision: 5 minutes before the exam

LAWS

- Kepler: ellipse, equal areas (L conserved), $T^2 \propto a^3$.
- $F = Gm_1m_2/r^2$, $G = 6.67 \times 10^{-11}$.
- Force inside a shell is 0.

HOW G CHANGES

- $g = GM/R^2$, maximum at the surface.
- Height: $g(1 - 2h/R)$; depth: $g(1 - d/R)$.
- Less at the equator, more at the poles.
- $g = 0$ at the centre.

POTENTIAL & ESCAPE

- $V = -GM/r$, $U = -GMm/r$.
- mgh only when $h \ll R$.
- $v_e = \sqrt{(2gR)} = 11.2$ km/s.

SATELLITES

- $v_o = \sqrt{(GM/r)}$; $v_e = \sqrt{2}\, v_o$.
- KE : PE : E = 1 : −2 : −1.
- Geostationary: 36,000 km, 24 h.
- Weightless = free fall; gravity is still there.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.