FREE NOTES · NCERT + JEE/NEET LEVEL

System of Particles & Rotational Motion

Centre of mass, torque, angular momentum, moment of inertia, τ = Iα, conservation of angular momentum aur rolling. Translational aur rotational motion ka side-by-side comparison ke saath.

01 Rigid body & types of motion
02 Centre of mass (COM)
03 Motion of centre of mass
04 Torque (moment of force)
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01 Rigid body & types of motion

Rigid body: aisi body jiske kisi bhi do points ke beech ki doori kabhi nahi badalti (shape fix). Asli me koi body poori rigid nahi, par chhote force pe hum aisa maan lete hain.

Motion	Matlab	Example
Pure translation	Har particle ki velocity same	Slope pe bina ghoome sarakta block
Pure rotation	Fixed axis ke chaaron taraf gol ghoomna	Pankha, ceiling fan, darwaza
Rotation + translation	Dono ek saath	Ludhakta pahiya, cylinder

02 Centre of mass (COM)

Wo point jahan body ka **poora mass ikattha maana** ja sakta hai motion samajhne ke liye. Zaroori nahi ki COM body ke andar ho (ring, bangle me centre khaali hai).

PARTICLES KE LIYE

$$X = \Sigma\, m_i x_i \,/\, \Sigma\, m_i$$

Same tarah Y aur Z. Do particles: COM bhaari wale ke paas, $m_1 r_1 = m_2 r_2$.

CONTINUOUS BODY

$$X = (1/M) \int x\, dm$$

Uniform aur symmetric body ka COM uske geometric centre pe hota hai.

Body (uniform)	COM ki position
Rod (length L)	Beech me, L/2
Semicircular ring (radius R)	Centre se 2R/π
Semicircular disc	Centre se 4R/(3π)
Hemisphere (solid)	Base se 3R/8
Hollow hemisphere	Base se R/2
Solid cone (height h)	Base se h/4
Triangle plate	Centroid (medians ka milan)

HISSA KAAT DIYA (CAVITY)

Poori body ko M aur kate hisse ko **negative mass** (−m) maan ke formula lagao: $X = (M\, x_1 - m\, x_2)/(M - m)$.

03 Motion of centre of mass

$$V_{cm} = \Sigma\, m_i v_i / M,\ A_{cm} = \Sigma\, m_i a_i / M$$

$$F_{ext} = M\, A_{cm},\ P_{total} = M\, V_{cm}$$

SABSE ZAROORI BAAT

Andar ke forces (internal) COM ki motion nahi badal sakte. Agar $\mathbf{F_{ext} = 0}$ toh V_{cm} constant. Isliye **hawa me phata bomb**: tukde idhar-udhar, par unka COM wahi parabola pe chalta rehta hai jis pe bomb chal raha tha.

Naav pe chalta aadmi: aadmi aage chale toh naav peeche khiskti hai, taaki system ka COM same jagah rahe: $m(x - X) = M\, X$ type ka sawaal.

04 Torque (moment of force)

Force kisi body ko ghumane ki jitni koshish karta hai, wo **torque** hai. Darwaza hinge ke paas dhakelo toh mushkil, door se dhakelo toh aasaan.

$$\tau = r \times F,\ |\tau| = r\, F \sin\theta$$

r = axis se force ke point tak, θ = r aur F ke beech angle. r sin θ = **moment arm** (axis se force ki line ki perpendicular doori). Unit N·m (joule nahi likhte).



Lamba lever ⇒ zyada torque

Couple: do barabar aur ulte forces jo ek line pe na hon. Net force 0, par torque = F × d (d = dono forces ke beech ki perpendicular doori). Sirf ghumata hai, aage nahi badhata (steering wheel, nal ki tonti).

05 Angular momentum

$$L = r \times p,\ |L| = r\, p \sin\theta = m\, v\, r_{\perp}$$

Rigid body ke liye: $L = I\, \omega$

$$\tau = dL/dt$$

Ye rotation ka "Newton's 2nd law" hai (F = dp/dt jaisa). Unit kg·m²/s = J·s.

SEEDHI LINE ME CHALTI PARTICLE

Kisi point ke hisaab se uska L = m v d (d = point se line ki perpendicular doori), aur ye **constant** rehta hai.

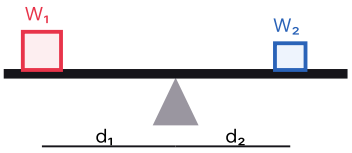
06 Equilibrium of a rigid body

DO SHARTEN

(1) Translational: $\Sigma F = 0$. (2) Rotational: kisi bhi point ke hisaab se $\Sigma \tau = 0$.

Principle of moments (lever): clockwise torque = anticlockwise torque. $\text{Load} \times \text{load arm} = \text{Effort} \times \text{effort arm}$. **Mechanical advantage** = load/effort = effort arm/load arm.

Centre of gravity: jahan poore weight ka torque zero ho. Uniform g me CG = COM. Bahut oonchi building me g badalta hai, toh thoda fark.



Balance: $W_1 d_1 = W_2 d_2$

07 Moment of inertia (I)

Rotation me **"inertia"**: body ghoomne ka kitna virodh karti hai. Ye sirf mass pe nahi, **mass axis se kitni door phaila hai** us pe depend karta hai.

$$I = \Sigma m r^2, \; I = \int r^2 \, dm$$

Unit kg·m². Radius of gyration k: $I = M k^2$ (aisi doori jahan poora mass rakhne pe same I mile).

Body	Axis	I
Ring (thin)	Centre se, plane ke ⊥	MR^2
Ring	Diameter	$MR^2/2$
Disc (solid)	Centre se, plane ke ⊥	$MR^2/2$
Disc	Diameter	$MR^2/4$
Hollow cylinder	Apni axis	MR^2
Solid cylinder	Apni axis	$MR^2/2$
Solid sphere	Diameter	$2MR^2/5$
Hollow sphere (shell)	Diameter	$2MR^2/3$
Rod (length L)	Beech se, ⊥	$ML^2/12$
Rod	Ek sire se, ⊥	$ML^2/3$
Rectangular plate (a × b)	Centre se, plane ke ⊥	$M(a^2 + b^2)/12$

KIS PE DEPEND KARTA HAI

Mass, mass ka distribution (shape), aur axis ki position. Angular velocity pe nahi. Same mass aur radius: ring ka I disc se zyada (sab mass kinare pe).

08 Parallel & perpendicular axis theorems

PARALLEL AXIS

$$I = I_{cm} + M\,d^2$$

Kisi bhi body ke liye. d = COM wali axis se nayi parallel axis ki doori. Example: rod sire se = $ML^2/12 + M(L/2)^2 = ML^2/3$.

PERPENDICULAR AXIS

$$I_z = I_x + I_y$$

Sirf **flat (2D) bodies** ke liye. x, y body ke plane me, z plane ke ⊥. Example: disc ka diameter wala I = $(MR^2/2)/2 = MR^2/4$.

DHYAN DO

Parallel axis me pehli axis **COM se guzarni chahiye**. Do aisi axes jo COM se nahi guzarti, unke beech seedha Md^2 nahi jod sakte.

09 Rotational kinematics & dynamics

KINEMATICS (A CONSTANT)

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$v = \omega r$, $a_t = \alpha r$, $a_c = \omega^2 r$.

DYNAMICS

$$\tau = I\,\alpha$$

Rotational KE $= \frac{1}{2} I\,\omega^2$

Work $= \tau\,\theta$, Power $= \tau\,\omega$

$W = \Delta K_{rot}$

PULLEY WALA SAWAAL

Mass M, radius R ki pulley pe rassi lipti, rassi pe m latka. $mg - T = ma$, $TR = I\alpha$, $a = \alpha R \Rightarrow$ disc pulley ($I = MR^2/2$) ke liye $a = mg/(m + M/2)$.

10 Conservation of angular momentum

STATEMENT

Agar **external torque = 0**, toh $L = I\omega = \text{constant} \Rightarrow I_1\omega_1 = I_2\omega_2$.

- **Ice skater / ballet dancer:** haath andar kheeche ⇒ I kam ⇒ ω badh jaata (tez ghoomti).
- **Diver:** hawa me pair modkar gol hota hai ⇒ zyada somersault.
- **Ghoomti kursi pe aadmi** dumbbells phailaye ⇒ dheema.
- Earth agar sikud jaaye (radius kam) toh din chhota ho jaayega.

DHYAN DO

L conserved hai par rotational KE nahi: $K = L^2/(2I)$. I kam karne pe K badhta hai (skater haath kheenchne me khud work karti hai).

11 Rolling motion (JEE/NEET)

Pahiya bina fisle ludhakta hai: $v_{cm} = \omega R$. Zameen ko chhoone wala point us pal **ruka hua** hai, top point $2v_{cm}$ se chalta hai.

ROLLING KI ENERGY

$$K = \tfrac{1}{2}Mv^2\,(1 + k^2/R^2)$$

Translational + rotational. k^2/R^2: ring 1, disc/cylinder ½, solid sphere 2/5, hollow sphere 2/3.

INCLINE PE LUDHAKNA (H, Θ)

$v = \sqrt{(2gh\,/\,(1 + k^2/R^2))}$

$a = g \sin\theta\,/\,(1 + k^2/R^2)$

Jiska k^2/R^2 kam, wo pehle neeche pahunchega.

RACE KA ORDER (EK SAATH CHHODA)

Solid sphere (pehla) → disc/solid cylinder → hollow sphere → ring (aakhri). Mass aur radius se fark nahi padta.

12 Linear vs rotational (ek table me)

Linear	Rotational
Displacement x	Angle θ
Velocity v	Angular velocity ω
Acceleration a	Angular acceleration α
Mass m	Moment of inertia I
Force $F = ma$	Torque $\tau = I\alpha$
Momentum $p = mv$	Angular momentum $L = I\omega$
$F = dp/dt$	$\tau = dL/dt$
KE = $\frac{1}{2}mv^2$	KE = $\frac{1}{2}I\omega^2$
Work = $F\ s$	Work = $\tau\ \theta$
Power = $F\ v$	Power = $\tau\ \omega$

13 JEE Extra + Tricks

JEE LEVEL

- Centre se axis ke MOI: ring MR^2, disc $MR^2/2$, hollow sphere $\frac{2}{3}MR^2$, solid sphere $\frac{2}{5}MR^2$, rod $ML^2/12$ (sire pe $ML^2/3$).
- Bina phisle rolling: KE = $\frac{1}{2}mv^2(1 + k^2/R^2)$; incline pe $a = g\sin\theta/(1 + k^2/R^2)$. Race (sabse tez pehle): solid sphere > disc > hollow sphere > ring.
- Centre of mass: semicircular ring $2R/\pi$, semicircular disc $4R/3\pi$, solid hemisphere $3R/8$, hollow hemisphere $R/2$, solid cone base se $h/4$.
- Net external torque zero ho to angular momentum conserved: $I\omega$ = constant (skater haath andar kare to tez ghoome).
- Theorems: parallel axis $I = I_{cm} + Md^2$; perpendicular axis $I_z = I_x + I_y$ (sirf planar).

14 Solved numericals (g = 10 m/s²)

Q1. 2 kg aur 3 kg ke particles x = 0 aur x = 5 m pe hain. COM?
$X = (2\times0 + 3\times5)/5 = 3$ m. **Ans: x = 3 m (bhaari wale ke paas)**

Q2. 20 N ka force spanner pe 0.25 m door 90° pe laga. Torque?
$\tau = rF\sin 90° = 0.25 \times 20 = 5$ N·m. **Ans: 5 N·m**

Q3. 2 kg, 0.5 m radius wali disc ka I centre se aur kinare ki tangent (plane ke ⊥) axis se?
$I_{cm} = \frac{1}{2} \times 2 \times 0.25 = 0.25$ kg·m². Kinare se: $0.25 + 2 \times 0.25 = 0.75$ kg·m². **Ans: 0.25, 0.75 kg·m²**

Q4. Flywheel (I = 4 kg·m²) pe 20 N·m torque. α aur 5 s baad ω (rest se)?
$\alpha = \tau/I = 5$ rad/s². $\omega = 25$ rad/s. **Ans: 5 rad/s², 25 rad/s**

Q5. Skater ka I = 6 kg·m², ω = 2 rad/s. Haath kheench ke I = 2 kg·m². Naya ω aur KE ka ratio?
$\omega = 6 \times 2/2 = 6$ rad/s. $K_1 = \frac{1}{2} \times 6 \times 4 = 12$ J, $K_2 = \frac{1}{2} \times 2 \times 36 = 36$ J ⇒ 3 guna. **Ans: 6 rad/s, KE 3 guna**

Q6. Solid sphere 7 m oonche incline se ludhakta hai (bina fisle). Neeche speed?
$v = \sqrt{(2gh/(1 + 2/5))} = \sqrt{(140/1.4)} = \sqrt{100} = 10$ m/s. **Ans: 10 m/s**

Q7. 1 m ki rod (2 kg) ek sire se ghoom rahi hai, ω = 3 rad/s. Rotational KE?
$I = ML^2/3 = 2/3$. $K = \frac{1}{2} \times (2/3) \times 9 = 3$ J. **Ans: 3 J**

15 Common mistakes

- COM ko hamesha body ke andar maan lena (ring me nahi hota).
- Perpendicular axis theorem ko 3D body (sphere) pe lagana.
- Parallel axis me COM se na guzarne wali axis se shuru karna.
- Rolling me sirf ½Mv² lena, ½Iω² bhool jaana.
- L conserve hai toh KE bhi conserve maan lena.
- Torque ki unit joule likhna (N·m likho).

16 Formula sheet

Centre of mass	$X = \Sigma mx/M$, $F_{ext} = MA_{cm}$
Torque	$\tau = r \times F = rF \sin\theta = I\alpha$
Angular momentum	$L = r \times p = I\omega$, $\tau = dL/dt$
Moment of inertia	$I = \Sigma mr^2 = Mk^2$
Parallel / perpendicular	$I = I_{cm} + Md^2$, $I_z = I_x + I_y$
Ring, disc, solid sphere, rod (centre)	MR^2, $MR^2/2$, $2MR^2/5$, $ML^2/12$
Rotational KE, work, power	$\frac{1}{2}I\omega^2$, $\tau\theta$, $\tau\omega$
Conservation	$I_1\omega_1 = I_2\omega_2$ $(\tau_{ext} = 0)$
Rolling KE	$\frac{1}{2}Mv^2(1 + k^2/R^2)$
Rolling down incline	$a = g \sin\theta/(1 + k^2/R^2)$

17 Practice questions

1. 1 m ki rod ke sire pe 1 kg aur 3 kg rakhe hain (rod ka mass ignore). COM pehle mass se kitni door?
2. Ring (M, R) ka I uski kisi tangent (plane me) ke hisaab se?
3. Ring aur disc same incline se ludhakte hain. Kaun pehle?
4. Earth ka radius aadha ho jaaye (mass same), din kitne ghante ka?
5. Pulley (disc, 2 kg) pe rassi se 1 kg latka. Acceleration?

ANSWERS

1) 0.75 m · 2) $MR^2/2 + MR^2 = 3MR^2/2$ · 3) Disc · 4) I ¼ ⇒ ω 4 guna ⇒ 6 ghante · 5) a = 10/(1 + 1) = 5 m/s²

18 Quick revision: exam se pehle 5 minute

COM

- X = Σmx/M; bhaari ke paas.
- Internal forces COM nahi hila sakte.
- Semicircular ring 2R/π, disc 4R/3π.
- Cavity: negative mass.

TORQUE & L

- τ = rF sin θ = Iα = dL/dt.
- L = Iω; τ = 0 ⇒ $I_1\omega_1 = I_2\omega_2$.
- Equilibrium: ΣF = 0, Στ = 0.

MOMENT OF INERTIA

- Ring MR^2, disc $MR^2/2$, sphere $2MR^2/5$, shell $2MR^2/3$.
- Rod: centre $ML^2/12$, end $ML^2/3$.
- $I = I_{cm} + Md^2$; $I_z = I_x + I_y$ (flat).

ROLLING

- $v = \omega R$; neeche wala point ruka.
- $K = \frac{1}{2}Mv^2(1 + k^2/R^2)$.
- Race: sphere > disc > shell > ring.

Doubt hai? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.