FREE NOTES · NCERT + JEE/NEET LEVEL

System of Particles & Rotational Motion

Centre of mass, torque, angular momentum, moment of inertia, τ = Iα, conservation of angular momentum and rolling, with a side-by-side comparison of translational and rotational motion.

01 Rigid body & types of motion
02 Centre of mass (COM)
03 Motion of the centre of mass
04 Torque (moment of force)
05 Angular momentum
06 Equilibrium of a rigid body
07 Moment of inertia (I)
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01 Rigid body & types of motion

Rigid body: a body in which the distance between any two points never changes (fixed shape). No real body is perfectly rigid, but for small forces we treat it as one.

Motion	Meaning	Example
Pure translation	Every particle has the same velocity	A block sliding down a slope without turning
Pure rotation	Turning about a fixed axis	Ceiling fan, a door
Rotation + translation	Both together	A rolling wheel or cylinder

02 Centre of mass (COM)

The point where the **whole mass can be taken as concentrated** to describe the motion. The COM need not lie inside the body (a ring or bangle has an empty centre).

FOR PARTICLES

$$X = \Sigma\, m_i x_i \,/\, \Sigma\, m_i$$

Similarly Y and Z. Two particles: COM is nearer the heavier one, $m_1 r_1 = m_2 r_2$.

CONTINUOUS BODY

$$X = (1/M) \int x\, dm$$

For a uniform symmetric body the COM is at its geometric centre.

Body (uniform)	Position of COM
Rod (length L)	Middle, L/2
Semicircular ring (radius R)	2R/π from the centre
Semicircular disc	4R/(3π) from the centre
Solid hemisphere	3R/8 from the base
Hollow hemisphere	R/2 from the base
Solid cone (height h)	h/4 from the base
Triangular plate	Centroid (where medians meet)

PART REMOVED (CAVITY)

Treat the full body as M and the removed part as a **negative mass** (−m): $X = (M\, x_1 - m\, x_2)/(M - m)$.

03 Motion of the centre of mass

$$V_{cm} = \Sigma\, m_i v_i / M,\ A_{cm} = \Sigma\, m_i a_i / M$$

$$F_{ext} = M\, A_{cm},\ P_{total} = M\, V_{cm}$$

THE KEY POINT

Internal forces cannot change the motion of the COM. If $\mathbf{F_{ext} = 0}$, V_{cm} is constant. So for a **bomb exploding in mid-air**, the fragments scatter but their COM continues along the same parabola the bomb was following.

Man walking on a boat: as the man walks forward the boat moves back so that the system's COM stays put: problems of the type $m(x - X) = M\, X$.

04 Torque (moment of force)

How strongly a force tries to rotate a body is its **torque**. Pushing a door near the hinge is hard; pushing far from it is easy.

$$\tau = r \times F,\ |\tau| = r\, F \sin\theta$$

r = from the axis to the point of force, θ = angle between r and F. r sin θ = **moment arm** (perpendicular distance from the axis to the line of force). Unit N·m (not written as joule).



A longer lever ⇒ more torque

Couple: two equal and opposite forces not on the same line. Net force is 0, but torque = F × d (d = perpendicular distance between them). It only rotates, never translates (steering wheel, tap).

05 Angular momentum

$$L = r \times p,\ |L| = r\, p \sin\theta = m\, v\, r_{\perp}$$

For a rigid body: $L = I\, \omega$

$$\tau = dL/dt$$

This is the rotational form of Newton's 2nd law (like F = dp/dt). Unit kg·m²/s = J·s.

A PARTICLE MOVING IN A STRAIGHT LINE

About a point, its L = m v d (d = perpendicular distance from the point to the line), and it stays **constant**.

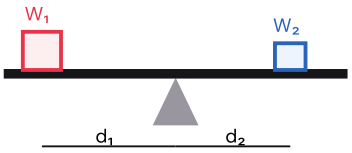
06 Equilibrium of a rigid body

TWO CONDITIONS

(1) Translational: $\Sigma F = 0$. (2) Rotational: $\Sigma \tau = 0$ about any point.

Principle of moments (lever): clockwise torque = anticlockwise torque. $\text{Load} \times \text{load arm} = \text{Effort} \times \text{effort arm}$. **Mechanical advantage** = load/effort = effort arm/load arm.

Centre of gravity: the point about which the total torque of weight is zero. In uniform g, CG = COM. For very tall buildings g varies, so they differ slightly.



Balance: $W_1d_1 = W_2d_2$

07 Moment of inertia (I)

The **"inertia" of rotation**: how much a body resists a change in rotation. It depends not just on mass but on **how far the mass is spread from the axis**.

$$I = \Sigma\, m\, r^2,\ I = \int r^2\, dm$$

Unit kg·m². Radius of gyration k: $I = M\, k^2$ (the distance at which all the mass would give the same I).

Body	Axis	I
Ring (thin)	Through centre, ⊥ to plane	MR^2
Ring	Diameter	$MR^2/2$
Disc (solid)	Through centre, ⊥ to plane	$MR^2/2$
Disc	Diameter	$MR^2/4$
Hollow cylinder	Own axis	MR^2
Solid cylinder	Own axis	$MR^2/2$
Solid sphere	Diameter	$2MR^2/5$
Hollow sphere (shell)	Diameter	$2MR^2/3$
Rod (length L)	Through middle, ⊥	$ML^2/12$
Rod	Through one end, ⊥	$ML^2/3$
Rectangular plate (a × b)	Through centre, ⊥ to plane	$M(a^2 + b^2)/12$

DEPENDS ON

Mass, how the mass is distributed (shape), and the position of the axis. Not on angular velocity. Same mass and radius: a ring has more I than a disc (all its mass is at the rim).

08 Parallel & perpendicular axis theorems

PARALLEL AXIS

$$I = I_{cm} + M\,d^2$$

For any body. d = distance between the COM axis and the new parallel axis. Example: rod about its end = $ML^2/12 + M(L/2)^2 = ML^2/3$.

PERPENDICULAR AXIS

$$I_z = I_x + I_y$$

Only for **flat (2D) bodies**. x and y lie in the plane, z is perpendicular. Example: disc about a diameter = $(MR^2/2)/2 = MR^2/4$.

NOTE

In the parallel axis theorem, the first axis **must pass through the COM**. You cannot directly add Md^2 between two axes that don't pass through the COM.

09 Rotational kinematics & dynamics

KINEMATICS (CONSTANT A)

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$v = \omega r$, $a_t = \alpha r$, $a_c = \omega^2 r$.

DYNAMICS

$$\tau = I\,\alpha$$

Rotational KE $= \frac{1}{2} I\,\omega^2$

Work $= \tau\,\theta$, Power $= \tau\,\omega$

$W = \Delta K_{rot}$

PULLEY PROBLEM

A string wound on a pulley of mass M, radius R, with mass m hanging. $mg - T = ma$, $TR = I\alpha$, $a = \alpha R$ ⇒ for a disc pulley ($I = MR^2/2$), $a = mg/(m + M/2)$.

10 Conservation of angular momentum

STATEMENT

If **external torque = 0**, then $L = I\omega = \text{constant} \Rightarrow I_1\omega_1 = I_2\omega_2$.

- **Ice skater / ballet dancer:** pulls arms in ⇒ I decreases ⇒ ω increases (spins faster).
- **Diver:** tucks the body in mid-air ⇒ more somersaults.
- **Man on a rotating stool** stretches out dumbbells ⇒ slows down.
- If the Earth shrank (smaller radius), the day would get shorter.

NOTE

L is conserved but rotational KE is not: $K = L^2/(2I)$. Reducing I increases K (the skater does work pulling her arms in).

11 Rolling motion (JEE/NEET)

A wheel rolling without slipping: $v_{cm} = \omega R$. The point touching the ground is **momentarily at rest**; the top point moves at $2v_{cm}$.

ENERGY OF ROLLING

$$K = \tfrac{1}{2}Mv^2\,(1 + k^2/R^2)$$

Translational + rotational. k^2/R^2: ring 1, disc/cylinder ½, solid sphere 2/5, hollow sphere 2/3.

ROLLING DOWN AN INCLINE (H, Θ)

$$v = \sqrt{(2gh\,/\,(1 + k^2/R^2))}$$

$$a = g\sin\theta\,/\,(1 + k^2/R^2)$$

The body with smaller k^2/R^2 reaches the bottom first.

RACE ORDER (RELEASED TOGETHER)

Solid sphere (first) → disc/solid cylinder → hollow sphere → ring (last). Mass and radius don't matter.

12 Linear vs rotational (in one table)

Linear	Rotational
Displacement x	Angle θ
Velocity v	Angular velocity ω
Acceleration a	Angular acceleration α
Mass m	Moment of inertia I
Force $F = ma$	Torque $\tau = I\alpha$
Momentum $p = mv$	Angular momentum $L = I\omega$
$F = dp/dt$	$\tau = dL/dt$
$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$
Work $= F\,s$	Work $= \tau\,\theta$
Power $= F\,v$	Power $= \tau\,\omega$

13 JEE Extra + Tricks

JEE LEVEL

- MOI about axis through centre: ring MR^2, disc $MR^2/2$, hollow sphere $\frac{2}{3}MR^2$, solid sphere $\frac{2}{5}MR^2$, rod $ML^2/12$ (about end $ML^2/3$).
- Rolling without slipping: KE = $\frac{1}{2}mv^2(1 + k^2/R^2)$; acceleration on incline $a = g\sin\theta/(1 + k^2/R^2)$. Race order (fastest first): solid sphere > disc > hollow sphere > ring.
- Centre of mass: semicircular ring $2R/\pi$, semicircular disc $4R/3\pi$, solid hemisphere $3R/8$, hollow hemisphere $R/2$, solid cone $h/4$ from base.
- Angular momentum conserved when net external torque is zero: $I\omega$ = constant (ice skater pulls arms in, spins faster).
- Theorems: parallel axis $I = I_{cm} + Md^2$; perpendicular axis $I_z = I_x + I_y$ (planar bodies only).

14 Solved numericals (g = 10 m/s²)

Q1. Particles of 2 kg and 3 kg are at x = 0 and x = 5 m. Find the COM.
$X = (2\times0 + 3\times5)/5 = 3$ m. **Ans: x = 3 m (nearer the heavier one)**

Q2. A 20 N force acts at 90° on a spanner, 0.25 m from the nut. Torque?
$\tau = rF\sin 90° = 0.25 \times 20 = 5$ N·m. **Ans: 5 N·m**

Q3. A 2 kg disc of radius 0.5 m: find I about its centre, and about a tangent at the rim perpendicular to the plane.
$I_{cm} = \frac{1}{2} \times 2 \times 0.25 = 0.25$ kg·m². At the rim: $0.25 + 2 \times 0.25 = 0.75$ kg·m². **Ans: 0.25, 0.75 kg·m²**

Q4. A flywheel (I = 4 kg·m²) gets a torque of 20 N·m. Find α and ω after 5 s (from rest).
$\alpha = \tau/I = 5$ rad/s². $\omega = 25$ rad/s. **Ans: 5 rad/s², 25 rad/s**

Q5. A skater has I = 6 kg·m², ω = 2 rad/s. Pulling her arms in makes I = 2 kg·m². New ω and ratio of KE?
$\omega = 6 \times 2/2 = 6$ rad/s. $K_1 = \frac{1}{2} \times 6 \times 4 = 12$ J, $K_2 = \frac{1}{2} \times 2 \times 36 = 36$ J ⇒ 3 times. **Ans: 6 rad/s, KE triples**

Q6. A solid sphere rolls without slipping down a 7 m high incline. Speed at the bottom?
$v = \sqrt{(2gh/(1 + 2/5))} = \sqrt{(140/1.4)} = \sqrt{100} = 10$ m/s. **Ans: 10 m/s**

Q7. A 1 m, 2 kg rod rotates about one end at ω = 3 rad/s. Rotational KE?
$I = ML^2/3 = 2/3$. $K = \frac{1}{2} \times (2/3) \times 9 = 3$ J. **Ans: 3 J**

15 Common mistakes

- Assuming the COM is always inside the body (not for a ring).
- Applying the perpendicular axis theorem to a 3D body (sphere).
- Starting the parallel axis theorem from an axis not through the COM.
- Taking only $\frac{1}{2}Mv^2$ for rolling and forgetting $\frac{1}{2}I\omega^2$.
- Assuming KE is conserved because L is conserved.
- Writing the unit of torque as joule (write N·m).

16 Formula sheet

Centre of mass	$X = \Sigma mx/M$, $F_{ext} = MA_{cm}$
Torque	$\tau = r \times F = rF \sin\theta = I\alpha$
Angular momentum	$L = r \times p = I\omega$, $\tau = dL/dt$
Moment of inertia	$I = \Sigma mr^2 = Mk^2$
Parallel / perpendicular	$I = I_{cm} + Md^2$, $I_z = I_x + I_y$
Ring, disc, solid sphere, rod (centre)	MR^2, $MR^2/2$, $2MR^2/5$, $ML^2/12$
Rotational KE, work, power	$\frac{1}{2}I\omega^2$, $\tau\theta$, $\tau\omega$
Conservation	$I_1\omega_1 = I_2\omega_2$ $(\tau_{ext} = 0)$
Rolling KE	$\frac{1}{2}Mv^2(1 + k^2/R^2)$
Rolling down an incline	$a = g \sin\theta/(1 + k^2/R^2)$

17 Practice questions

1. 1 kg and 3 kg masses are at the ends of a 1 m rod (rod mass negligible). Distance of COM from the 1 kg mass?
2. I of a ring (M, R) about a tangent in its plane?
3. A ring and a disc roll down the same incline. Which arrives first?
4. If the Earth's radius halved (same mass), how long would a day be?
5. A 1 kg mass hangs from a string on a 2 kg disc pulley. Acceleration?

ANSWERS

1) 0.75 m · 2) $MR^2/2 + MR^2 = 3MR^2/2$ · 3) Disc · 4) I becomes ¼ ⇒ ω 4 times ⇒ 6 hours · 5) a = 10/(1 + 1) = 5 m/s²

18 Quick revision: 5 minutes before the exam

COM

- $X = \Sigma mx/M$; nearer the heavier mass.
- Internal forces can't move the COM.
- Semicircular ring $2R/\pi$, disc $4R/3\pi$.
- Cavity: negative mass.

TORQUE & L

- $\tau = rF \sin\theta = I\alpha = dL/dt$.
- $L = I\omega$; $\tau = 0 \Rightarrow I_1\omega_1 = I_2\omega_2$.
- Equilibrium: $\Sigma F = 0$, $\Sigma\tau = 0$.

MOMENT OF INERTIA

- Ring MR^2, disc $MR^2/2$, sphere $2MR^2/5$, shell $2MR^2/3$.
- Rod: centre $ML^2/12$, end $ML^2/3$.
- $I = I_{cm} + Md^2$; $I_z = I_x + I_y$ (flat).

ROLLING

- $v = \omega R$; bottom point at rest.
- $K = \frac{1}{2}Mv^2(1 + k^2/R^2)$.
- Race: sphere > disc > shell > ring.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.