FREE NOTES · NCERT + JEE/NEET LEVEL

Work, Energy & Power

When work is done, kinetic and potential energy, the work-energy theorem, energy conservation, springs, power, collisions and the vertical circle. Explained simply, with diagrams and solved numericals.

01 Work
02 Work by a variable force
03 Kinetic energy
04 Work-energy theorem
05 Potential energy & conservative forces
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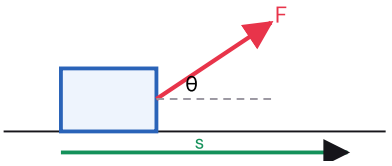
01 Work

In physics, **work is done when a force displaces a body** and part of the force is along the displacement. Push a wall all day without moving it and the work done is 0.

FORMULA

$$W = F \cdot s = F\, s \cos\theta$$

θ = angle between force and displacement. Unit: joule (J) = N·m. Work is a scalar. 1 erg = 10^{-7} J, 1 eV = 1.6×10^{-19} J, 1 kWh = 3.6×10^{6} J.



Only the F cos θ part does work

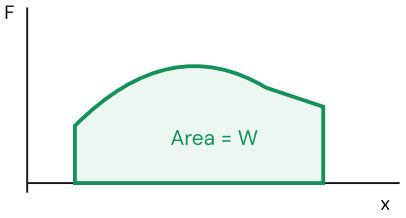
Angle θ	Work	Example
$0° \le \theta < 90°$	Positive	Gravity on a falling object; a horse pulling a cart
$\theta = 90°$	Zero	A porter walking horizontally with a load; centripetal force in circular motion; normal force
$90° < \theta \le 180°$	Negative	Friction; gravity on a ball thrown up; brakes

02 Work by a variable force

If the force changes with position, add up the work over small pieces:

$$W = \int F\, dx$$

Area under the F–x graph = work. Area below the x-axis is negative work.



Force–displacement graph

03 Kinetic energy

Energy a body has because of its motion: $K = \frac{1}{2} m v^2$. Always positive (or 0).

KE AND MOMENTUM

$$K = p^2 / (2m),\ p = \sqrt{(2mK)}$$

Same KE: the lighter body has less momentum. Same momentum: the lighter body has more KE.

PERCENTAGE SHORTCUTS

- Double v ⇒ 4 times K. Double p ⇒ 4 times K.
- Small % change: % change in K ≈ 2 × (% change in v). p up 10% ⇒ K up 21% (exact: $1.1^2 = 1.21$).

04 Work-energy theorem

STATEMENT

The **total work done by all forces** on a body equals the change in its kinetic energy.

$W_{net} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

It holds for every kind of force (constant or variable, conservative or not). It saves long calculations: no need to find acceleration.

Proof (constant force): $v^2 - u^2 = 2as \Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas = Fs = W$.

NOTE

W_{net} must include the work of **all** forces: gravity, friction, normal, applied. Taking only the applied force is wrong.

05 Potential energy & conservative forces

Energy stored because of position or shape = **potential energy (U)**. Near the Earth's surface: $U = mgh$.

	Conservative force	Non-conservative force
Work depends on	Only initial and final positions, not the path	The path taken
Work in a closed loop	Zero	Not zero
Potential energy	Can be defined	Cannot
Examples	Gravity, spring force, electrostatic force	Friction, air resistance, viscous force

FORCE AND PE

$$F = -dU/dx$$

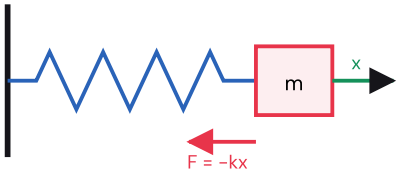
Work by a conservative force = $-\Delta U$. Equilibrium when $F = 0$, i.e. $dU/dx = 0$. U minimum ⇒ **stable**, U maximum ⇒ **unstable**, U constant ⇒ **neutral** equilibrium.

06 Energy of a spring

Stretch or compress a spring by x and it pulls back: $F = -kx$ (Hooke's law). k = spring constant (N/m); larger k means a stiffer spring.

$$U = \frac{1}{2}\,k\,x^2$$

Work to stretch from x_1 to $x_2 = \frac{1}{2}k(x_2^2 - x_1^2)$. U is positive whether stretched or compressed.



The spring force always points toward the natural length

COMBINING SPRINGS

Series: $1/k = 1/k_1 + 1/k_2$ (softer). Parallel: $k = k_1 + k_2$ (stiffer). Cut a spring into n equal pieces: each has k' = nk.

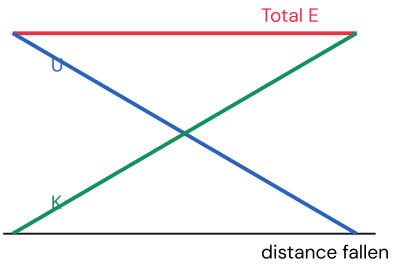
07 Conservation of mechanical energy

STATEMENT

If only **conservative forces** do work, $K + U = \text{constant}$. Energy changes form but is not lost.

Ball falling from height H: at the top all PE (mgH), midway part KE part PE, at the bottom all KE (½mv² = mgH ⇒ v = √(2gH)).

With friction: $K_1 + U_1 = K_2 + U_2 + W_{\text{friction}}$. The energy lost to friction becomes heat.



U falls, K rises, total stays the same

Other forms of energy: heat, chemical, electrical, nuclear ($E = mc^2$). Total energy is always conserved.

08 Power

FORMULA

$$P = dW/dt = F \cdot v$$

Average power = total work / total time. Unit watt (W) = J/s. 1 horsepower (hp) = 746 W. kWh is a unit of **energy**, not power.

EXAMPLE

A pump lifting m kg of water to height h every t seconds: $P = mgh/t$. Efficiency = useful output / input × 100%.

09 Collisions

In a collision large internal forces act for a very short time, so **momentum is always conserved**. Kinetic energy is not always.

	Elastic	Inelastic	Perfectly inelastic
Momentum	Conserved	Conserved	Conserved
Kinetic energy	Conserved	Decreases	Maximum loss
e (coefficient of restitution)	1	0 < e < 1	0 (bodies stick)

COEFFICIENT OF RESTITUTION

$$e = (v_2 - v_1)/(u_1 - u_2)$$

Speed of separation / speed of approach.

1D ELASTIC COLLISION

$$v_1 = ((m_1 - m_2)u_1 + 2m_2u_2)/(m_1 + m_2)$$

$$v_2 = ((m_2 - m_1)u_2 + 2m_1u_1)/(m_1 + m_2)$$

SPECIAL CASES (U_2 = 0)

- $m_1 = m_2$: velocities **exchange** (the first one stops).
- $m_1 \gg m_2$: heavy one continues at u_1, light one moves at $2u_1$.
- $m_1 \ll m_2$ (ball on a wall): ball rebounds at $-u_1$.

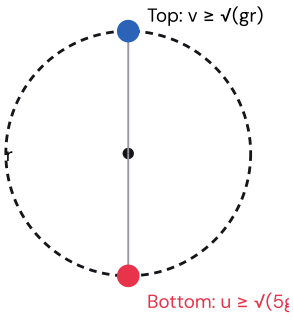
PERFECTLY INELASTIC

$$v = (m_1u_1 + m_2u_2)/(m_1 + m_2), \text{ KE loss} = m_1m_2(u_1 - u_2)^2/(2(m_1 + m_2))$$

Bouncing ball: dropped from height h, after n bounces it rises to $h_n = e^{2n}\,h$.

10 Vertical circular motion

Whirl a stone (or bucket) on a string in a vertical circle. Speed is greatest at the bottom and least at the top.



Condition to complete the circle

STRING CASE

Minimum speed at top: $v = \sqrt{(gr)}$

Minimum speed at bottom: $u = \sqrt{(5gr)}$

At the horizontal point: $\sqrt{(3gr)}$

$T_{bottom} - T_{top} = 6mg$

With a rod (instead of a string) speed at the top can be 0: √(4gr) at the bottom is enough.

Speed at the bottom u	What happens
$u \geq \sqrt{(5gr)}$	Completes the circle
$\sqrt{(2gr)} < u < \sqrt{(5gr)}$	String goes slack in the upper half; it falls as a projectile
$u \leq \sqrt{(2gr)}$	Oscillates in the lower half (like a pendulum)

11 JEE Extra + Tricks

JEE LEVEL

- Conservative force: $F = -dU/dx$. Equilibrium where $dU/dx = 0$; **stable** at U minimum, unstable at U maximum.
- Vertical circle (string): minimum speed at bottom $\sqrt{(5gr)}$, at top $\sqrt{(gr)}$; tension difference $T_{bottom} - T_{top} = 6mg$.
- Collisions: coefficient of restitution e = (separation speed)/(approach speed); elastic e = 1, perfectly inelastic e = 0.
- 1D elastic collision with equal masses: velocities **exchange**. Heavy hits light at rest: light moves with ~2u.
- Perfectly inelastic: KE loss = $\frac{1}{2} \cdot m_1 m_2/(m_1 + m_2) \cdot (u_1 - u_2)^2$.
- Power = F·v; for a vehicle pulling at constant power, $F = P/v$.

12 Solved numericals (g = 10 m/s²)

Q1. A box is pulled 10 m by a 50 N force at 60° to the horizontal. Work done?
$W = 50 \times 10 \times \cos 60° = 250$ J. **Ans: 250 J**

Q2. A 2 kg body speeds up from 3 m/s to 5 m/s. Net work?
$W = \frac{1}{2} \times 2 \times (25 - 9) = 16$ J. **Ans: 16 J**

Q3. Momentum increases by 20%. By what % does KE increase?
$K \propto p^2 \Rightarrow (1.2)^2 = 1.44 \Rightarrow$ 44% increase. **Ans: 44%**

Q4. A spring with k = 200 N/m is compressed 10 cm. Stored energy? With what speed will a 0.5 kg ball leave it?
$U = \frac{1}{2} \times 200 \times 0.01 = 1$ J. $\frac{1}{2} \times 0.5 \times v^2 = 1 \Rightarrow v = 2$ m/s. **Ans: 1 J, 2 m/s**

Q5. A 1 kg block slides down a smooth 5 m high slope onto a rough floor (μ = 0.5). How far does it go before stopping?
KE at bottom = mgh = 50 J. Work against friction = $\mu mg\, d = 5d \Rightarrow d = 10$ m. **Ans: 10 m**

Q6. A motor lifts 100 kg through 20 m in 10 s. Power?
$P = mgh/t = 100 \times 10 \times 20/10 = 2000$ W ≈ 2.7 hp. **Ans: 2 kW**

Q7. A 2 kg ball at 3 m/s sticks to a stationary 1 kg ball. Common velocity and KE loss?
$v = 2 \times 3/3 = 2$ m/s. KE before 9 J, after $\frac{1}{2} \times 3 \times 4 = 6$ J ⇒ loss 3 J. **Ans: 2 m/s, 3 J**

Q8. A stone on a 1 m string must complete a vertical circle. Minimum speed at the bottom?
$u = \sqrt{(5gr)} = \sqrt{50} \approx 7.07$ m/s. **Ans: ≈ 7.1 m/s**

13 Common mistakes

- Thinking "effort means work". Zero displacement or force $\perp$ displacement $\Rightarrow$ zero work.
- Using only the applied force in the work-energy theorem and forgetting gravity and friction.
- Assuming KE is conserved in every collision. Only in elastic ones.
- Treating kWh as a unit of power. It is a unit of energy.
- Mixing up the rod and string cases in the vertical circle.
- Leaving x in cm in spring PE. Convert to metres.

14 Formula sheet

Work	$W = Fs\cos\theta,\ W = \int F\,dx$
Kinetic energy	$K = \tfrac{1}{2}mv^2 = p^2/2m$
Work-energy theorem	$W_{net} = \Delta K$
Gravitational PE	$U = mgh$
Spring	$F = -kx,\ U = \tfrac{1}{2}kx^2$
Force from PE	$F = -dU/dx$
Power	$P = dW/dt = F\cdot v,\ 1\text{ hp} = 746\text{ W}$
Coefficient of restitution	$e = (v_2 - v_1)/(u_1 - u_2)$
Perfectly inelastic	$v = (m_1u_1 + m_2u_2)/(m_1 + m_2)$
Bounce height	$h_n = e^{2n}\,h$
Vertical circle (string)	$v_{top} \geq \sqrt{(gr)},\ u_{bottom} \geq \sqrt{(5gr)}$

15 Practice questions

1. A porter carries a 20 kg load on his head and walks 50 m horizontally. Work done by gravity?
2. Two bodies with equal KE have masses in the ratio 1 : 4. Ratio of their momenta?
3. $F = (3x^2 + 2)$ N. Work done from $x = 0$ to 2 m?
4. A ball dropped from 10 m has $e = 0.5$. How high does it rise after the first bounce?
5. A car moves at 20 m/s with the engine applying 1000 N. Power?

ANSWERS

1) 0 (force $\perp$ displacement) · 2) $p \propto \sqrt{m} \Rightarrow 1:2$ · 3) $[x^3 + 2x]_0^2 = 12$ J · 4) $e^2h = 2.5$ m · 5) 20 kW

16 Quick revision: 5 minutes before the exam

WORK & ENERGY

- $W = Fs\cos\theta$; $\theta = 90° \Rightarrow 0$.
- Area under F–x = work.
- $W_{net} = \Delta K$ (all forces).
- $K = p^2/2m$.

PE & CONSERVATION

- Conservative: path-independent, 0 in a loop.
- $F = -dU/dx$; U min ⇒ stable.
- Spring $U = \frac{1}{2}kx^2$.
- Only conservative ⇒ K + U constant.

POWER & COLLISIONS

- $P = F \cdot v$; 1 hp = 746 W.
- Momentum always conserved; KE only in elastic.
- Equal masses, elastic ⇒ velocities exchange.
- $h_n = e^{2n}h$.

VERTICAL CIRCLE

- String: top $\sqrt{(gr)}$, bottom $\sqrt{(5gr)}$.
- $T_{bottom} - T_{top} = 6mg$.
- Rod: $\sqrt{(4gr)}$ at the bottom is enough.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.