FREE NOTES · NCERT + JEE/NEET LEVEL

Laws of Motion

From Newton's three laws to friction, pulleys and banked roads, explained simply. Every topic has a formula box, diagram, solved numericals and common mistakes.

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01 What is force?

A **force** is a push or a pull that changes (or tries to change) a body's **speed, direction or shape**. SI unit: **newton (N)**. 1 N = 1 kg·m/s².

- **Contact forces:** act when two objects touch, e.g. normal force, friction, tension, a push.
- **Non-contact (field) forces:** act without touching, e.g. gravity, magnetic force, electric force.

Force is a **vector**, so two forces are not simply added. We take their **vector sum**.

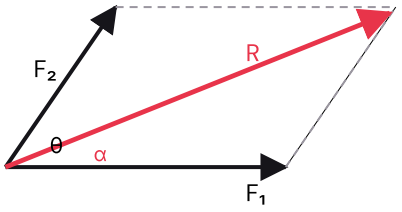
FORMULA: RESULTANT OF TWO FORCES

$$R = \sqrt{(F_1^2 + F_2^2 + 2F_1F_2 \cos\theta)}$$

$$\tan\alpha = F_2 \sin\theta / (F_1 + F_2 \cos\theta)$$

θ = angle between the forces, α = angle of the resultant with F_1.

Special case $F_1 = F_2 = F$: $R = 2F\cos(\theta/2)$, α = θ/2.



Parallelogram law: R is the diagonal

02 Inertia & Newton's 1st law

NCERT: ARISTOTLE'S FALLACY VS GALILEO

Aristotle believed a force must keep acting for a body to keep moving. **Galileo** showed with inclined planes that a ball rolling down one slope climbs the opposite slope to *almost the same height*. The flatter the second slope, the farther the ball travels. On a perfectly flat, smooth surface it would **keep moving forever**. The ball stops only because of friction. This is where the idea of inertia comes from.

Inertia is a body's inability to change its own state of rest or uniform motion. **More mass means more inertia.** Inertia depends only on mass, not on shape or speed.

NEWTON'S 1ST LAW (LAW OF INERTIA)

If the **net external force is zero**, a body at rest stays at rest and a moving body keeps moving with the **same velocity** (same speed, same direction).

Type of inertia	Meaning	Everyday example
Inertia of rest	A body at rest cannot start moving by itself	When a bus starts suddenly we fall backward; beating a carpet removes dust
Inertia of motion	A moving body cannot stop by itself	When a bus brakes suddenly we fall forward; a long jumper runs before jumping
Inertia of direction	A body cannot change direction by itself	We lean outward when a bus turns; mud flies off a spinning wheel along the tangent

REMEMBER

The 1st law *defines* force (force is what changes a body's state). The 2nd law tells us how much force.

03 Momentum & Newton's 2nd law

Linear momentum $p = m\,v$ measures the "quantity of motion" in a body. It is a vector along the velocity. Unit: kg·m/s.

NEWTON'S 2ND LAW

The rate of change of momentum equals the applied **net external force** and takes place in the direction of the force.

FORMULAS

$$F = dp/dt = d(mv)/dt$$

Situation	Force
Constant mass, changing velocity (most problems)	$F = m\,(dv/dt) = m\,a$
Constant velocity, changing mass (sand falling on a conveyor belt)	$F = v\,(dm/dt)$
Both change (rocket)	$F = m\,a + v\,(dm/dt)$
Rocket thrust (gas leaves at relative speed u)	$F_{thrust} = u\,(dm/dt)$

COMMON MISTAKE

$F = ma$ holds only when mass is constant. Don't forget the dm/dt term in rocket or conveyor problems.

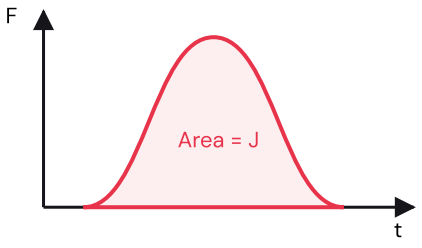
04 Impulse

When a large force acts for a very short time (bat on ball, hammer blow), we look at the product of force and time. This is called **impulse (J)**.

IMPULSE–MOMENTUM THEOREM

$$J = F_{avg} \cdot \Delta t = \Delta p = m\,v - m\,u$$

Area under the F–t graph = impulse. Unit: N·s (= kg·m/s)



Force–time graph

IN REAL LIFE

- A cricketer **pulls the hands back** while catching: Δt increases, so force and pain decrease.
- Jumping onto a mattress or sand hurts less: stopping time is longer.
- Car **airbags and crumple zones** work on the same principle.

05 Newton's 3rd law

STATEMENT

To every action there is an **equal and opposite reaction**. If A exerts a force on B, B exerts an equal force on A in the opposite direction: $F_{AB} = -F_{BA}$

- Action and reaction act on **different bodies**, so they never cancel each other.
- Both arise at the same instant; neither comes first.
- Examples: while walking, the foot pushes the ground back and the ground pushes us forward. A rocket pushes gas down and the gas pushes the rocket up. A swimmer pushes water back.

COMMON MISTAKE: "BOOK ON A TABLE"

The book's weight (Earth's pull) and the table's normal force are **NOT an action-reaction pair**. Both act on the same body (the book). The reaction to weight is the book pulling Earth upward. The reaction to the normal force is the book pressing down on the table.

06 Conservation of linear momentum

If the **net external force** on a system is zero, the system's total momentum stays constant. This follows from the 2nd and 3rd laws: internal forces cancel in pairs.

FORMULA

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Gun recoil: everything starts at rest, so $0 = m_bv_b + M_gV_g \Rightarrow V_g = -m_bv_b/M_g$ (minus = opposite direction)

Applications: recoil of a gun, explosion of a bomb (total momentum of fragments = 0), jumping from a boat to the shore, rockets.

07 Equilibrium & Lami's theorem

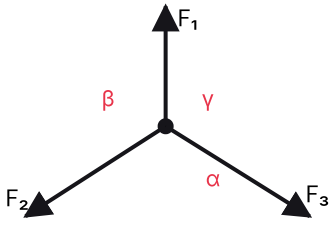
A body is in **equilibrium** when the net force on it is zero: $\Sigma F_x = 0$, $\Sigma F_y = 0$. It is either at rest or moving with constant velocity.

LAMI'S THEOREM

Three concurrent forces keeping a body in equilibrium:

$$F_1/\sin\alpha = F_2/\sin\beta = F_3/\sin\gamma$$

Each angle is the one **opposite** that force (between the other two forces).

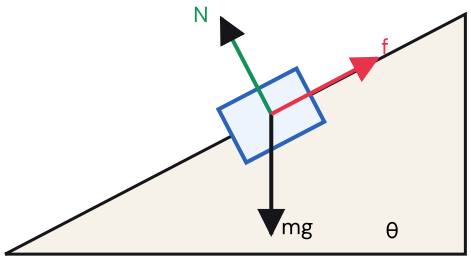


α = angle between F_2 and F_3 (opposite F_1)

08 Free Body Diagram (FBD): the key to every problem

Most Laws of Motion problems are solved by a correct FBD. Steps:

1. Draw the body in question **separately**, as a dot or a box.
2. Mark **all forces acting on it**: weight mg (down), normal N (perpendicular to the surface), tension T (along the string), friction f (opposing motion), any applied force.
3. Do not draw forces the body exerts on *other* bodies.
4. Choose convenient axes (on an incline: along and perpendicular to the slope).
5. Write $\Sigma F = m\,a$ along each axis (0 in equilibrium).



Block on an incline: mg, N, friction f (up the slope, since the block tends to slide down)

COMPONENTS ON AN INCLINE

Along the slope: $mg \sin\theta$

Perpendicular to slope: $mg \cos\theta$

$N = mg \cos\theta$

Smooth incline: $a = g \sin\theta$

With friction (sliding down): $a = g(\sin\theta - \mu_k \cos\theta)$

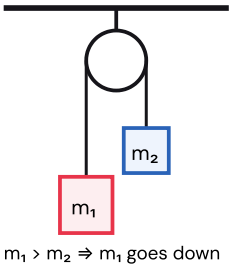
Common forces at a glance

Force	Direction	When it acts
Weight mg	Always straight down	On every body
Normal N	Perpendicular to the surface, away from it	When a body presses a surface. Not always equal to mg!
Tension T	Along the string, away from the body (pulls)	Same everywhere in a light string
Friction f	Opposite to relative motion (or its tendency)	On rough surfaces
Spring force	Back toward natural length	$F = -k\,x$

09 Connected bodies & pulleys

Trick: first treat the **whole system** as one body to find acceleration, then draw the FBD of one body to find tension or contact force.

(a) Atwood machine

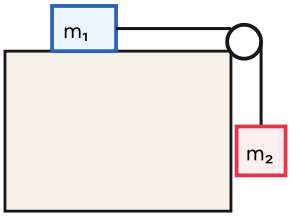


$m_1 > m_2 \Rightarrow m_1$ goes down

$$a = (m_1 - m_2)\, g \,/\, (m_1 + m_2)$$

$$T = 2\, m_1\, m_2\, g \,/\, (m_1 + m_2)$$

(b) Block on table + hanging block



Smooth table:

$$a = m_2\, g \,/\, (m_1 + m_2)$$

$$T = m_1\, m_2\, g \,/\, (m_1 + m_2)$$

With friction μ: $a = (m_2 - \mu\, m_1)\, g \,/\, (m_1 + m_2)$

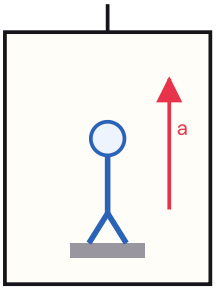
(c) Two blocks in contact, pushed by force F

$$a = F \,/\, (m_1 + m_2) \quad \text{contact force on } m_2 = m_2\, F \,/\, (m_1 + m_2)$$

10 Apparent weight in a lift

A weighing machine always reads the **normal force N**, not the true weight. In a lift, N changes:

State of the lift	Reading N
At rest or constant velocity	$N = mg$
Accelerating up (a)	$N = m\,(g + a)$
Accelerating down (a)	$N = m\,(g - a)$
Free fall (cable breaks, a = g)	$N = 0$ (weightless)



In a lift accelerating upward you feel "heavier"

11 Friction

Friction is the force that **opposes relative motion (or its tendency)** between two surfaces in contact. It comes from surface roughness and molecular attraction.

Type	When	Value
Static friction	Body has not started moving	Self-adjusting: from 0 up to $f_{s,max} = \mu_s N$, only as much as needed
Kinetic friction	Body is sliding	$f_k = \mu_k N$ (nearly constant)
Rolling friction	Wheel is rolling	Smallest: that's why we use wheels and ball bearings

REMEMBER THE ORDER

$\mu_{rolling} < \mu_k < \mu_s$

That's why a heavy cupboard is hard to start moving but easier once it moves.

Laws of friction

- Limiting friction is proportional to the normal force: $f \propto N$.
- It is (nearly) independent of the area of contact.
- It depends on the nature of the surfaces (material, roughness).

ANGLE OF FRICTION (Λ)

$$\tan \lambda = \mu_s$$

Angle between N and the resultant of N and f_{max}.

ANGLE OF REPOSE (Θ_R)

$$\tan \theta_r = \mu_s$$

Incline angle at which a block is just about to slide. $\lambda = \theta_r$.

COMMON MISTAKE

Static friction is not always $\mu_s N$! If the limit is 10 N and you push with 4 N, friction is only 4 N and the body doesn't move.

FRICTION IS ALSO NECESSARY

Walking, vehicle brakes and writing with a pen all depend on friction. Ways to reduce it: lubricants (oil), ball bearings, polishing, streamlined shapes.

12 Circular motion: level road & banked road

To move in a circle, a body needs a force toward the centre, the **centripetal force**: $F_c = m v^2/r$. It is not a new force; tension, friction, normal force or gravity provides it.

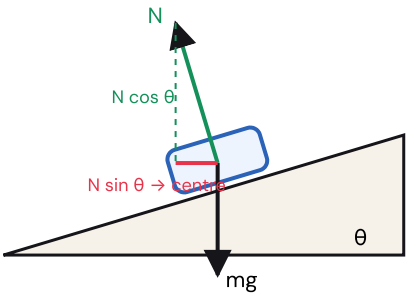
(a) Car on a level (flat) road

Only **friction** provides the centripetal force:

$$m\,v^2/r \le \mu_s\, m\, g$$

$$v_{max} = \sqrt{(\mu_s\, r\, g)}$$

Independent of mass.



N cos θ = mg, N sin θ = mv²/r ⇒ tan θ = v²/rg

(b) Banked road

The road is tilted at angle θ so the horizontal component of the normal force provides the centripetal force.

Without friction (optimum speed):

$$\tan\theta = v^2 / (r\, g)$$

Maximum speed with friction μ:

$$v_{max} = \sqrt{(\, r\, g\, (\mu + \tan\theta) / (1 - \mu \tan\theta)\,)}$$

A **cyclist** leans inward on a turn at angle $\tan\theta = v^2/(r\, g)$, by the same logic.

13 JEE Extra + Tricks

JEE LEVEL

- Atwood machine: $a = (m_1 - m_2)g/(m_1 + m_2)$, $T = 2m_1m_2g/(m_1 + m_2)$.
- Non-inertial frame: add **pseudo force** $-ma$ (opposite to frame's acceleration). Lift accelerating up: apparent weight $m(g + a)$; free fall: weightless.
- Angle of repose: $\tan\theta = \mu_s$. Block on incline slides if $\tan\theta > \mu$.
- Banked road: $\tan\theta = v^2/(rg)$ (no friction); with friction $v_{max}^2 = rg(\mu + \tan\theta)/(1 - \mu\tan\theta)$.
- Constraint trick: length of string is constant ⇒ sum of (displacements × number of string segments) = 0.
- Spring cut into n equal parts: each has $k' = nk$.

14 Solved numericals (g = 10 m/s²)

Q1. Two forces of 3 N and 4 N act perpendicular to each other. Find the resultant and its angle with the 3 N force.
$R = \sqrt{(3^2 + 4^2 + 2 \cdot 3 \cdot 4 \cdot \cos 90°)} = \sqrt{25} = 5$ N. $\tan \alpha = 4/3 \Rightarrow \alpha \approx 53°$. **Ans: 5 N, 53°**

Q2. A 150 g ball hits a bat at 30 m/s and returns at 20 m/s in the opposite direction. Contact time is 0.01 s. Find the average force by the bat.
Take the incoming direction as negative: $\Delta p = m(v - u) = 0.15 \times (20 - (-30)) = 0.15 \times 50 = 7.5$ N·s. $F = \Delta p / \Delta t = 7.5/0.01$. **Ans: 750 N**

Q3. A 20 g bullet is fired at 400 m/s from a 4 kg gun. Find the recoil velocity of the gun.
Momentum conservation: $0 = 0.02 \times 400 + 4 \times V \Rightarrow V = -8/4 = -2$ m/s. **Ans: 2 m/s, opposite to the bullet**

Q4. An Atwood machine has blocks of 5 kg and 3 kg. Find acceleration and tension.
$a = (5 - 3) \times 10 / (5 + 3) = 2.5$ m/s². $T = 2 \times 5 \times 3 \times 10 / 8 = 37.5$ N. **Ans: 2.5 m/s², 37.5 N**

Q5. A 60 kg student stands on a weighing machine in a lift. Find the reading when the lift (i) accelerates up at 2 m/s², (ii) accelerates down at 2 m/s², (iii) falls freely.
(i) $N = 60(10 + 2) = 720$ N, (ii) $N = 60(10 - 2) = 480$ N, (iii) $N = 0$. **Ans: 720 N, 480 N, 0**

Q6. A block slides down a 30° incline with $\mu_k = 0.2$. Find its acceleration.
$a = g(\sin 30° - \mu \cos 30°) = 10(0.5 - 0.2 \times 0.866) = 10 \times 0.327$. **Ans: ≈ 3.27 m/s²**

Q7. A 4 kg block on a smooth table is tied by a string over a pulley to a hanging 1 kg block. Find a and T.
$a = 1 \times 10 / (4 + 1) = 2$ m/s². $T = m_1 a = 4 \times 2 = 8$ N (check: 1×10 − 8 = 1×2 ✓). **Ans: 2 m/s², 8 N**

Q8. A flat road has a turn of radius 50 m and $\mu_s = 0.5$. Find the maximum safe speed.
$v = \sqrt{(\mu r g)} = \sqrt{(0.5 \times 50 \times 10)} = \sqrt{250} \approx 15.8$ m/s ≈ 57 km/h. **Ans: ≈ 15.8 m/s**

Q9. At what angle should a 100 m radius turn be banked for a speed of 20 m/s (no friction)?
$\tan \theta = v^2/(r g) = 400/1000 = 0.4 \Rightarrow \theta \approx 21.8°$. **Ans: ≈ 22°**

Q10. Blocks of 2 kg and 3 kg touch each other on a smooth floor. The 2 kg block is pushed with 20 N. Find the contact force on the 3 kg block.
$a = 20/5 = 4$ m/s². Contact force = 3 × 4 = 12 N. **Ans: 12 N**

15 Common mistakes (where marks are lost)

- **N is not always mg**: on an incline it is $mg\cos\theta$, in a lift $m(g \pm a)$, more if pushed from above.
- Putting action and reaction on the same body. They always act on different bodies.
- Taking static friction directly as $\mu_s N$, when it is only as much as needed.
- Forgetting the sign of direction in impulse problems (a rebounding ball reverses velocity).
- Adding centripetal force as a separate force in the FBD. It is the job of an existing force.
- Mixing units: convert grams to kg and km/h to m/s (km/h × 5/18 = m/s).

16 One-page formula sheet

Resultant of two forces	$R = \sqrt{(F_1^2 + F_2^2 + 2F_1F_2 \cos\theta)}$
Momentum	$p = m\,v$
Newton's 2nd law	$F = dp/dt = m\,a$ (constant mass)
Variable mass / conveyor	$F = v\,(dm/dt)$
Rocket thrust	$F = u\,(dm/dt)$
Impulse	$J = F\,\Delta t = \Delta p = m(v - u)$
Momentum conservation	$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
Gun recoil	$V = -\,m\,v\,/\,M$
Lami's theorem	$F_1/\sin\alpha = F_2/\sin\beta = F_3/\sin\gamma$
Smooth incline	$a = g\sin\theta$, $N = mg\cos\theta$
Rough incline (sliding down)	$a = g(\sin\theta - \mu_k\cos\theta)$
Atwood machine	$a = (m_1-m_2)g/(m_1+m_2)$, $T = 2m_1m_2g/(m_1+m_2)$
Block on table + hanging block	$a = m_2g/(m_1+m_2)$, $T = m_1m_2g/(m_1+m_2)$
Lift (apparent weight)	$N = m(g + a)$ up, $m(g - a)$ down, 0 free fall
Static / kinetic friction	$f_s \le \mu_sN$, $f_k = \mu_kN$
Angle of repose / friction	$\tan\theta_r = \tan\lambda = \mu_s$
Centripetal force	$F = m\,v^2/r$
Max speed, level road	$v = \sqrt{(\mu\,r\,g)}$
Banked road (no friction)	$\tan\theta = v^2/(r\,g)$
Banked road, max speed	$v = \sqrt{(rg(\mu + \tan\theta)/(1 - \mu\tan\theta))}$
Cyclist leaning	$\tan\theta = v^2/(r\,g)$

17 Practice questions (try yourself)

1. The velocity of a 2 kg body changes from 4 m/s to 10 m/s in 3 s. Find the force applied.
2. What minimum horizontal force is needed to move a 10 kg block on the floor? (μ_s = 0.4)
3. Gas leaves a rocket at 500 m/s relative speed at a rate of 2 kg/s. Find the thrust.
4. A 1000 kg car takes a turn of radius 20 m at 10 m/s. How much friction force is needed?

5. A horse pulls a cart and the cart pulls the horse back equally (3rd law). How does the system move forward?
6. A lift falls with acceleration g. What is the apparent weight of a 50 kg man?

ANSWERS

1) 4 N · 2) 40 N · 3) 1000 N · 4) 5000 N · 6) 0

5) The cart's backward pull on the horse and the ground's forward push on the horse's feet are different forces. The horse pushes the ground backward, and the ground's reaction (friction) pushes the system forward. When this ground force exceeds the friction on the cart, the system moves forward.

18 Quick revision: 5 minutes before the exam

NEWTON'S LAWS

- 1st law: net force 0 ⇒ constant velocity. Inertia ∝ mass.
- 2nd law: F = dp/dt; constant mass ⇒ F = ma.
- 3rd law: action = - reaction, on **different bodies**.
- Impulse = FΔt = Δp. Increase Δt ⇒ smaller force.
- Net external force 0 ⇒ total momentum conserved.

FORCES & FBD

- One body, all forces on it, then ΣF = ma on each axis.
- Incline: N = mg cos θ, along slope mg sin θ.
- Lift: N = m(g ± a); in free fall N = 0.
- Light string: same T everywhere.
- Lami: F/sin(opposite angle) is constant.

FRICTION

- Static: 0 to $\mu_s N$ (self-adjusting).
- Kinetic: $\mu_k N$. Order: rolling < kinetic < static.
- tan(angle of repose) = μ_s.
- Independent of contact area.

CIRCULAR MOTION

- Centripetal = mv^2/r, not a new force.
- Level road: $v_{max} = \sqrt{(\mu rg)}$.
- Banked, no friction: $\tan\theta = v^2/rg$.
- Cyclist's lean: $\tan\theta = v^2/rg$.

Top 5 exam traps	Correct thinking
Weight and normal force = action-reaction	No, both act on the same body
N = mg always	Only on a flat surface with no extra vertical force
Static friction = $\mu_s N$	That is only its maximum value
Rebounding ball: Δp = m(v - u) directly	Use direction signs: magnitude Δp = m(v + u)
Acceleration is 0 in uniform circular motion	No, it is v^2/r toward the centre

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the 30-question mock test on this chapter.