FREE NOTES · NCERT + JEE/NEET LEVEL

Motion in a Plane

Vectors se lekar projectile, circular motion aur river-boat / baarish-chhatri wale relative velocity sawaalon tak. Har topic ke saath diagram, shortcut aur solved numericals.

01 Scalar vs vector
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01 Scalar vs vector

	Scalar	Vector
Kya batata hai	Sirf magnitude (kitna)	Magnitude + direction
Jodna	Seedha number jodo	Triangle / parallelogram law se
Examples	Mass, time, distance, speed, work, energy, temperature	Displacement, velocity, acceleration, force, momentum

SPECIAL VECTORS

- **Equal vectors:** same magnitude, same direction (kahin bhi rakho).
- **Negative vector:** same magnitude, ulti direction: −A.
- **Null (zero) vector:** magnitude 0, direction kuch bhi. A + (−A) = 0.
- **Unit vector:** magnitude 1, sirf direction batata hai: Â = A/|A|.

DHYAN DO

Current ki direction hoti hai par ye **scalar** hai, kyunki ye vector ke rules (triangle law) se nahi judta. Vector banne ke liye direction ke saath vector addition ka rule bhi follow karna zaroori hai.

02 Vectors ko jodna

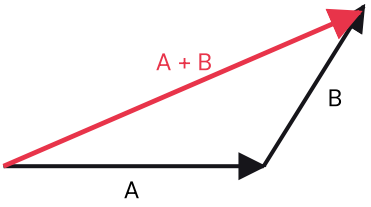
Triangle law: A ki nok pe B ki poonch rakho. A ki poonch se B ki nok tak ka arrow = A + B.

Parallelogram law: dono ko ek hi point se banao. Parallelogram ka diagonal = resultant.

MAGNITUDE & DIRECTION

$$R = \sqrt{(A^2 + B^2 + 2AB\cos\theta)}$$

$$\tan\alpha = B\sin\theta / (A + B\cos\theta)$$



Triangle law: nok se poonch (head to tail)

Angle θ	Resultant R
0° (same direction)	A + B (maximum)
90°	√(A² + B²)
180° (ulti direction)	|A − B| (minimum)
A = B, angle θ	2A cos(θ/2); θ = 120° pe R = A

RANGE RULE

$|A - B| \leq |A + B| \leq A + B$. Teen vectors ka sum zero tabhi ho sakta hai jab teesra pehle do ke sum ki range me ho (triangle ban sake).

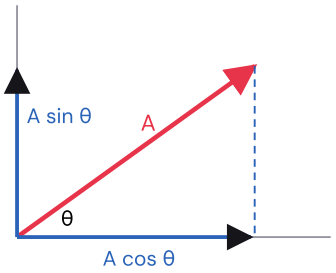
Subtraction: $A - B = A + (-B)$, magnitude $\sqrt{(A^2 + B^2 - 2AB\cos\theta)}$. Vector addition commutative (A + B = B + A) aur associative hai, subtraction commutative nahi.

03 Resolution & unit vectors

Kisi vector ko do perpendicular hisson me todna **resolution** hai. x-axis se angle θ ho toh:

$$A_x = A\cos\theta,\ A_y = A\sin\theta$$

$$A = \sqrt{(A_x^2 + A_y^2)},\ \tan\theta = A_y/A_x$$



A_x aur A_y milke wapas A banate hain

Unit vectors $\hat{i}, \hat{j}, \hat{k}$: x, y, z axis ki direction. Har vector: $A = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$, magnitude $\sqrt{(A_x^2 + A_y^2 + A_z^2)}$.

DIRECTION COSINES (3D)

Axes se angles α, β, γ: $\cos\alpha = A_x/A$, $\cos\beta = A_y/A$, $\cos\gamma = A_z/A$, aur $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

04 Dot & cross product

DOT (SCALAR) PRODUCT

$$A \cdot B = AB\cos\theta$$

$$= A_xB_x + A_yB_y + A_zB_z$$

$\hat{i}\cdot\hat{i} = 1$, $\hat{i}\cdot\hat{j} = 0$. Perpendicular ⇒ dot = 0. Example: work W = F · s.

CROSS (VECTOR) PRODUCT

$$|A \times B| = AB\sin\theta$$

Direction: right-hand rule (dono ke perpendicular). $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$; $\hat{i} \times \hat{i} = 0$. A × B = −B × A. Parallel ⇒ cross = 0. Example: torque τ = r × F.

Angle nikalne ke liye: $\cos\theta = (A \cdot B)/(AB)$. Component of A along B: $A\cos\theta = (A \cdot B)/B$.

05 2D me velocity & acceleration

Position vector $r = x\,\hat{i} + y\,\hat{j}$. Displacement $\Delta r = r_2 - r_1$.

$$v = dr/dt = v_x\,\hat{i} + v_y\,\hat{j},\ a = dv/dt = a_x\,\hat{i} + a_y\,\hat{j}$$

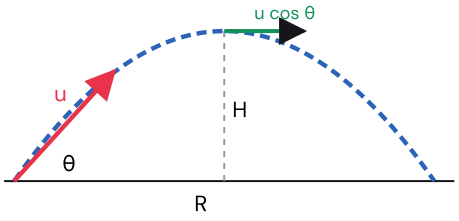
Velocity hamesha path ki **tangent** ki direction me hoti hai. Constant a ho toh har axis pe alag-alag: $v = u + at$, $r = r_0 + ut + \frac{1}{2}at^2$.

SABSE BADI BAAT

x aur y direction ki motions **ek doosre se independent** hain. Isi idea se projectile ke saare sawaal solve hote hain.

06 Projectile motion

Hawa me phenki cheez jis pe sirf gravity lag rahi ho. Horizontal me koi force nahi ⇒ horizontal velocity constant. Vertical me a = -g.



Sabse upar sirf horizontal velocity u cos θ bachti hai

GROUND SE GROUND

Time of flight: $T = 2u \sin\theta / g$

Max height: $H = u^2 \sin^2\theta / (2g)$

Range: $R = u^2 \sin 2\theta / g$

Path: $y = x \tan\theta - g x^2 / (2u^2 \cos^2\theta)$

Path parabola hai.

Yaad rakhne wale results	
Max range	$\theta = 45°$ pe, $R_{max} = u^2/g$, tab $H = R/4$
Same range	θ aur $(90° - \theta)$ pe range barabar (jaise $30°$ aur $60°$)
H aur R ka rishta	$\tan\theta = 4H/R$
H aur T	$H = gT^2/8$
Sabse upar	$v = u\cos\theta$, $a = g$ (neeche), velocity $\perp$ acceleration
Kinetic energy sabse upar	$K\cos^2\theta$ (K = shuru ki KE)

07 Horizontal projection (height h se seedha phenka)

Time: $T = \sqrt{(2h/g)}$ (u pe depend nahi karta!)

Range: $R = u\sqrt{(2h/g)}$

Zameen pe speed: $v = \sqrt{(u^2 + 2gh)}$, $\tan\beta = \sqrt{(2gh)}/u$ (horizontal se neeche)

CLASSIC SAWAAL

Ek goli horizontal chalaao aur usi waqt doosri goli usi oonchai se chhodo: **dono ek saath** zameen pe girengi, kyunki vertical motion dono ki same hai.

INCLINED PLANE (JEE)

Angle α wale incline pe, incline se θ angle pe phenka: $T = 2u\sin\theta/(g\cos\alpha)$, $R = 2u^2\sin\theta\cos(\theta + \alpha)/(g\cos^2\alpha)$ (upar ki taraf). Max range tab jab $\theta = 45° - \alpha/2$.

08 Uniform circular motion

Constant speed se gol ghoomna. Speed same hai par direction har pal badal rahi hai, isliye **acceleration hai**, centre ki taraf.

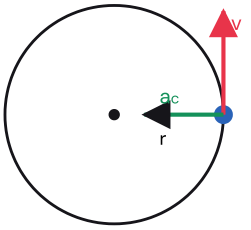
FORMULAS

Angular velocity: $\omega = d\theta/dt = 2\pi/T = 2\pi f$

$v = \omega r$

$$a_c = v^2/r = \omega^2 r$$

Centripetal acceleration hamesha centre ki taraf. T = ek chakkar ka time, f = 1/T.



v tangent pe, a_c centre ki taraf: $v \perp a$

DHYAN DO

Uniform circular motion me speed constant hai par **velocity aur acceleration dono badalte rehte hain** (direction ki wajah se). Ye accelerated motion hai.

09 Non-uniform circular motion

Agar speed bhi badal rahi ho toh do acceleration hote hain:

Tangential: $a_t = dv/dt = r\,\alpha$ (speed badalta hai)

Centripetal: $a_c = v^2/r$ (direction badalta hai)

$$a = \sqrt{(a_t^2 + a_c^2)}$$

α = angular acceleration (rad/s²). Angular equations bhi linear jaisi: $\omega = \omega_0 + \alpha t$, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, $\omega^2 = \omega_0^2 + 2\alpha\theta$.

10 Relative velocity in 2D

$v_{AB} = v_A - v_B$ (vector subtraction).

BAARISH AUR CHHATRI

Baarish v_r se seedhi gir rahi hai, aadmi v_m se chal raha hai. Aadmi ko baarish $v_{rm} = v_r - v_m$ se aati dikhti hai. Chhatri vertical se angle θ pe aage jhukao: $\tan\theta = v_m/v_r$.

RIVER-BOAT

River ki chaudai d, river speed v_r, boat ki speed (paani ke hisaab se) v_b.

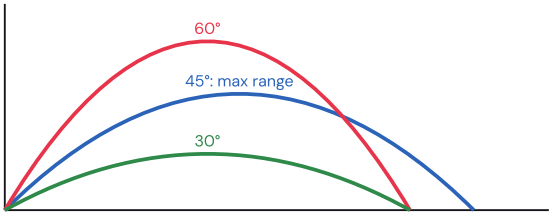
Kam se kam time: boat seedha paar ⇒ $t = d/v_b$, drift $= v_r d/v_b$

Seedha saamne pahunchna (zero drift): upstream angle $\sin\theta = v_r/v_b$, $t = d/\sqrt{(v_b^2 - v_r^2)}$

11 JEE Extra + Tricks

JEE LEVEL

- Projectile: $T = 2u\sin\theta/g$, $H = u^2\sin^2\theta/2g$, $R = u^2\sin2\theta/g$; 45° pe R max = u^2/g; $H_{max} = R_{max}/2$.
- Complementary angles (θ, 90° − θ) pe same range; $T_1T_2 = 2R/g$.
- Trajectory: $y = x\tan\theta - gx^2/(2u^2\cos^2\theta) = x\tan\theta\,(1 - x/R)$.
- Nadi paar: sabse kam samay ⇒ seedha perpendicular tairo, $t = d/v_s$; sabse chhota raasta (drift zero) ⇒ upstream $\sin\alpha = v_r/v_s$.
- Uniform circular motion: $a = v^2/r = \omega^2 r$ centre ki taraf; non-uniform me tangential $a_t = dv/dt$ bhi; net $a = \sqrt{(a_c^2 + a_t^2)}$.



30° and 60° give the same range (complementary angles)

Same speed se 30°, 45°, 60° pe projectiles

12 Solved numericals (g = 10 m/s²)

Q1. A = 3î + 4ĵ aur B = î − 2ĵ. |A + B| aur A · B?
A + B = 4î + 2ĵ ⇒ |A + B| = √20 = 2√5. A · B = 3 − 8 = −5. **Ans: 2√5, −5**

Q2. Do barabar forces F ka resultant bhi F hai. Unke beech angle?
F = 2F cos(θ/2) ⇒ cos(θ/2) = ½ ⇒ θ/2 = 60°. **Ans: 120°**

Q3. Ball 20 m/s se 30° pe phenki. T, H aur R?
T = 2 × 20 × ½/10 = 2 s. H = 400 × ¼/20 = 5 m. R = 400 × sin 60°/10 = 40 × 0.866 ≈ 34.6 m. **Ans: 2 s, 5 m, ≈ 34.6 m**

Q4. 80 m oonchi cliff se patthar 15 m/s se horizontal phenka. Kitni door girega aur kis speed se?
T = √(160/10) = 4 s. R = 15 × 4 = 60 m. v_y = 40 m/s ⇒ v = √(225 + 1600) = √1825 ≈ 42.7 m/s. **Ans: 60 m, ≈ 42.7 m/s**

Q5. Kisi projectile ki range uski max height ki 4 guna hai. Angle?
tan θ = 4H/R = 4H/4H = 1 ⇒ **Ans: 45°**

Q6. Patthar 1 m ki rassi se baandh ke 2 chakkar/second se ghumaya. Centripetal acceleration?
ω = 2π × 2 = 4π rad/s. a = ω²r = 16π² ≈ 158 m/s². **Ans: ≈ 158 m/s²**

Q7. Baarish 30 m/s se seedhi gir rahi hai, aadmi 10 m/s se east chal raha hai. Chhatri kaise pakde?
tan θ = 10/30 = 1/3 ⇒ θ ≈ 18.4° vertical se, east (aage) ki taraf jhuka ke. **Ans: ≈ 18° aage jhukao**

Q8. 400 m chaudi river 3 m/s se beh rahi hai, boat ki speed 5 m/s. (a) Kam se kam time? (b) Seedha saamne pahunchne me time?
(a) t = 400/5 = 80 s, drift = 3 × 80 = 240 m. (b) t = 400/√(25 − 9) = 400/4 = 100 s. **Ans: 80 s, 100 s**

13 Common mistakes

- Projectile me sabse upar velocity 0 maan lena. Sirf vertical part 0 hota hai, u cos θ bachta hai.
- Range formula (u² sin 2θ/g) ko height se phenkne pe bhi laga dena. Ye sirf ground-to-ground ke liye hai.
- Uniform circular motion me acceleration 0 maan lena.
- Vectors ke magnitudes seedhe jod dena: 3 N + 4 N hamesha 7 N nahi.
- Chhatri ko peeche jhukaana. Chalte waqt baarish saamne se aati dikhti hai, chhatri **aage** jhukti hai.
- Kam se kam time aur kam se kam drift wale river cases mix kar dena.

14 Formula sheet

Resultant	$R = \sqrt{(A^2 + B^2 + 2AB\cos\theta)}$
Components	$A_x = A\cos\theta$, $A_y = A\sin\theta$
Dot / cross	$A\cdot B = AB\cos\theta$, $\lvert A\times B\rvert = AB\sin\theta$
Projectile T, H, R	$2u\sin\theta/g$, $u^2\sin^2\theta/2g$, $u^2\sin 2\theta/g$
Max range	u^2/g at $45°$
Trajectory	$y = x\tan\theta - gx^2/(2u^2\cos^2\theta)$
Horizontal projection	$T = \sqrt{(2h/g)}$, $R = u\sqrt{(2h/g)}$
Circular motion	$v = \omega r$, $\omega = 2\pi/T$, $a_c = v^2/r = \omega^2 r$
Non-uniform	$a = \sqrt{(a_t^2 + a_c^2)}$
Rain-umbrella	$\tan\theta = v_m/v_r$
River: min time / zero drift	d/v_b / $d/\sqrt{(v_b^2 - v_r^2)}$

15 Practice questions

1. A = 2î + 3ĵ − k̂ aur B = î − ĵ + k̂ perpendicular hain? Check karo.
2. Projectile ki max height 20 m hai. Time of flight?
3. Kis angle pe phenkne se range aur max height barabar honge?
4. Car 36 km/h se 50 m radius ke mod pe hai. Centripetal acceleration?
5. Patthar 5 m oonchai se 10 m/s horizontal phenka. Range?

ANSWERS

1) A·B = 2 − 3 − 1 = −2 ≠ 0, perpendicular nahi · 2) T = √(8H/g) = 4 s · 3) tan θ = 4 ⇒ θ ≈ 76° · 4) 100/50 = 2 m/s² · 5) 10 × 1 = 10 m

16 Quick revision: exam se pehle 5 minute

VECTORS

- $R = \sqrt{(A^2 + B^2 + 2AB\cos\theta)}$.
- $|A - B| \le R \le A + B$.
- $A \cdot B = 0 \Rightarrow$ perpendicular; $A \times B = 0 \Rightarrow$ parallel.
- Barabar vectors 120° pe $\Rightarrow R = A$.

PROJECTILE

- Horizontal v constant, vertical $a = -g$.
- $T = 2u\sin\theta/g$, $H = u^2\sin^2\theta/2g$, $R = u^2\sin 2\theta/g$.
- 45° pe max range; θ aur 90° - θ same range.
- Horizontal phenka: $T = \sqrt{(2h/g)}$.

CIRCULAR

- $v = \omega r$, $a_c = v^2/r = \omega^2 r$.
- Uniform: speed same, velocity badalti.
- Non-uniform: $a = \sqrt{(a_t^2 + a_c^2)}$.

RELATIVE

- $v_{AB} = v_A - v_B$.
- Chhatri: $\tan\theta = v_m/v_r$, aage jhukao.
- River: min time d/v_b; zero drift $\sin\theta = v_r/v_b$.

Doubt hai? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.