FREE NOTES · NCERT + JEE/NEET LEVEL

Motion in a Straight Line

Distance, displacement, speed, velocity, acceleration, the three equations of motion, free fall, graphs and relative velocity. Shortcut formulas and solved numericals with every topic.

01 Distance vs displacement
02 Speed & velocity
03 Special cases of average speed (direct MCQs)
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01 Distance vs displacement

To describe motion, first choose a **reference point (origin)** and a direction. On a straight line one side is + and the other is −.

	Distance (path length)	Displacement
Meaning	The full path actually travelled	Straight change from initial to final position: $\Delta x = x_f - x_i$
Type	Scalar	Vector
Value	Always + (or 0)	Can be +, − or 0
From a graph	Total area under the v–t graph (all taken +)	Area under the v–t graph (above +, below −)

RULE

Distance ≥ |Displacement|. They are equal only when the body moves in one direction without turning back.

FOR ASSERTION-REASON QUESTIONS

- If displacement is 0, distance may or may not be 0 (a round trip).
- If distance is 0, displacement must be 0.
- If displacement is non-zero, distance must be non-zero.

PART OF A CIRCLE (ANGLE Θ)

$$\text{Distance} = R\,\theta$$

$$\text{Displacement} = 2R\sin(\theta/2)$$

Half round: distance πR, displacement 2R. Full round: distance 2πR, displacement 0.

02 Speed & velocity

AVERAGE

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time}}$$

INSTANTANEOUS

$$v = dx/dt$$

Slope of the x–t graph. A speedometer shows its magnitude.

- |Instantaneous velocity| = instantaneous speed: **always**.
- |Average velocity| = average speed only if the direction never changes. Otherwise average speed > |average velocity|.
- One lap of a circular track: average velocity is 0, average speed is not.

03 Special cases of average speed (direct MCQs)

Situation	Average speed
v_1 for time t_1, v_2 for time t_2	$(v_1t_1 + v_2t_2)/(t_1 + t_2)$
Distance x_1 at v_1, x_2 at v_2	$(x_1 + x_2)/(x_1/v_1 + x_2/v_2)$
Half the **time** at v_1, half at v_2	$(v_1 + v_2)/2$ (arithmetic mean)
Half the **distance** at v_1, half at v_2	$2v_1v_2/(v_1 + v_2)$ (harmonic mean)
Three equal distances at v_1, v_2, v_3	$3v_1v_2v_3/(v_1v_2 + v_2v_3 + v_3v_1)$
Half distance at v_1; remaining time split equally at v_2, v_3	$2v_1(v_2 + v_3)/(2v_1 + v_2 + v_3)$

COMMON MISTAKE

Writing $(v_1 + v_2)/2$ for the half-distance case. When distances are equal use the **harmonic mean**, which is always smaller than the arithmetic mean.

04 Acceleration

The rate of change of velocity. Unit m/s², a vector quantity.

FORMULAS

$$a_{avg} = (v_f - v_i)/\Delta t$$

$$a = dv/dt = d^2x/dt^2 = v\,(dv/dx)$$

Slope of the v–t graph = acceleration.

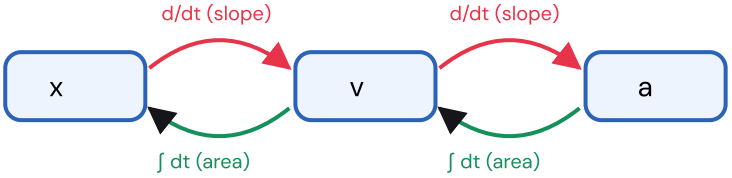
WILL SPEED INCREASE OR DECREASE?

- v and a in the **same direction** (0°): speed increases.
- v and a **opposite** (180°): speed decreases (retardation).
- v ⊥ a (90°): speed constant, only direction changes (circular motion).

NOTE

Negative acceleration does not always mean "slowing down". If the body is also moving in the − direction, negative acceleration makes it **faster**. Speed decreases when v and a have opposite signs.

05 Calculus: the golden rule



Differentiate (slope) to go forward, integrate (area) to come back

Given	How to solve
a = f(t)	$v = \int a\,dt$, then $x = \int v\,dt$
a = f(v)	$dv/dt = f(v) \Rightarrow \int dv/f(v) = \int dt$
a = f(x)	$v\,dv/dx = f(x) \Rightarrow \int v\,dv = \int f(x)\,dx$

WHEN IS ACCELERATION CONSTANT?

If $x \propto t^2$, or $v \propto t$, or $v \propto \sqrt{x}$, acceleration is constant. If x has t^3 or a higher power, acceleration changes.

06 Equations of motion (constant acceleration)

THE THREE EQUATIONS

$$v = u + a\,t$$

$$s = u\,t + \tfrac{1}{2}\,a\,t^2$$

$$v^2 = u^2 + 2\,a\,s$$

Bonus: $s = (u + v)\,t / 2$, $s = v\,t - \tfrac{1}{2}\,a\,t^2$. u = initial velocity, v = final, s = displacement.

DISTANCE IN THE N-TH SECOND

$$S_n = u + (a/2)(2n - 1)$$

The units look odd because this is the distance in "1 second" (a hidden $\Delta t = 1$ s).

WHEN TO USE

Only when acceleration is **constant**. For changing acceleration use calculus (section 05).

Derivation using calculus (NCERT)

$a = dv/dt \Rightarrow \int_u^v dv = a\int_0^t dt \Rightarrow v = u + at$. Then $v = dx/dt = u + at \Rightarrow x - x_0 = ut + \tfrac{1}{2}at^2$. And $a = v\,dv/dx \Rightarrow \int v\,dv = a\int dx \Rightarrow v^2 - u^2 = 2a(x - x_0)$.

07 Shortcut results

Situation	Result
From rest, constant a: distance in the 1st, 2nd, 3rd… second	$1 : 3 : 5 : 7 \ldots$ (odd numbers, Galileo)
From rest: total distance in 1, 2, 3 s	$1 : 4 : 9 \ (\propto t^2)$
From rest: time taken to cover equal successive distances	$1 : (\sqrt{2}-1) : (\sqrt{3}-\sqrt{2}) \ldots$
Velocity at the mid-point	$v_{mid} = \sqrt{((u^2+v^2)/2)}$
Stopping distance (speed u, retardation a)	$s = u^2/(2a) \propto u^2$
Rest → accelerate α → retard β → rest, total time T	$v_{max} = \alpha\beta T/(\alpha+\beta)$, $s = \frac{1}{2}\,\alpha\beta T^2/(\alpha+\beta)$

REACTION TIME (NCERT)

Until the driver presses the brake, the car keeps its old speed. Total stopping distance = $u \cdot t_r + u^2/(2a)$. Double the speed ⇒ 4 times the braking distance.

08 Motion under gravity (free fall)

Without air resistance every object falls with the same acceleration $g \approx 9.8\ m/s^2$, heavy or light.

SIGN CONVENTION

Taking up as +, $a = -g$ always (going up, coming down, and at the top). At the top, velocity is 0 but acceleration is not.

DROPPED FROM HEIGHT H (U = 0)

$t = \sqrt{(2H/g)}$, $v = \sqrt{(2gH)}$

Time to fall half the height $= T/\sqrt{2}$

THROWN UP FROM THE GROUND (SPEED U)

$H_{max} = u^2/(2g)$

$t_{up} = t_{down} = u/g$, $T = 2u/g$

It returns to the same point with the same speed u.

THROWN UP WITH U FROM HEIGHT H

$-H = u\,t - \frac{1}{2}\,g\,t^2$

$t = u/g + \sqrt{(u^2/g^2 + 2H/g)}$

WITH AIR RESISTANCE (RETARDATION A)

Up: $g + a$, down: $g - a$

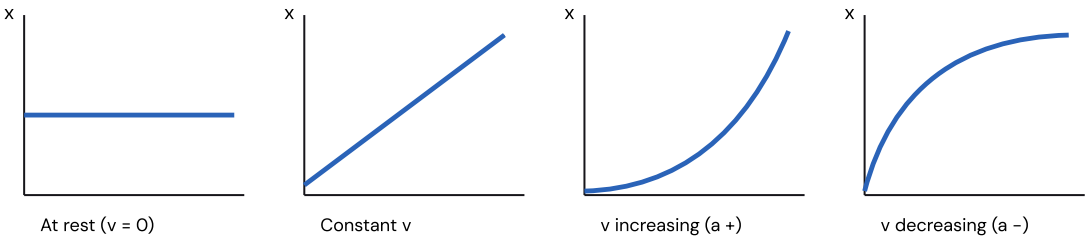
$t_{up}/t_{down} = \sqrt{((g-a)/(g+a))}$

Going up takes less time than coming down.

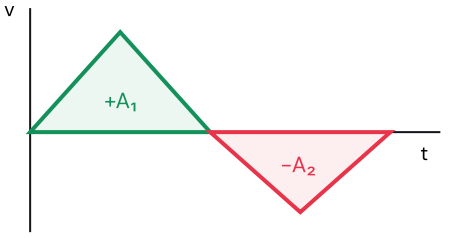
09 Graphs (x–t, v–t, a–t)

IN ONE LINE

Slope of x–t = velocity. **Slope** of v–t = acceleration. **Area** under v–t = displacement. **Area** under a–t = change in velocity.



Position–time graphs: the slope of the curve is the velocity



Displacement = $A_1 - A_2$, distance = $A_1 + A_2$

IMPOSSIBLE GRAPHS

- An x–t graph with two positions at the same time (vertical line or a graph bending back).
- Negative speed or distance.
- Sudden breaks (jumps) or sharp corners in x–t or v–t (infinite acceleration).

10 Relative velocity

Velocity of A relative to B: $v_{AB} = v_A - v_B$. That is how fast A appears to someone sitting in B.

Situation	Relative speed
Same direction	$\lvert v_A - v_B \rvert$ (approach slowly)
Opposite directions	$v_A + v_B$ (approach quickly)
Same velocity	0: each appears at rest to the other, the gap stays constant

TRAIN CROSSING TRICK

One train crossing another: distance = $L_1 + L_2$, speed = relative speed. Crossing a pole or a person: distance = only the train's length.

11 JEE Extra + Tricks

JEE LEVEL

- Distance in n-th second: $s_n = u + a(2n - 1)/2$.
- Galileo's odd-number rule: from rest with constant a, distances in equal successive intervals are 1 : 3 : 5 : 7 ...
- Stopping distance $\propto u^2$ (same braking): doubling speed $\Rightarrow$ 4× stopping distance.
- Ball thrown up with u: time up = u/g, max height $u^2/2g$, returns with same speed; time of flight 2u/g.
- Relative velocity: $v_{AB} = v_A - v_B$; two bodies meet when relative displacement = initial separation.
- Graph tricks: slope of x–t = v; slope of v–t = a; area under v–t = displacement; area under a–t = change in v.

12 Solved numericals ($g = 10\ m/s^2$)

Q1. A car covers half the distance at 40 km/h and half at 60 km/h. Average speed?
Harmonic mean: $2 \times 40 \times 60/(40 + 60) = 4800/100$. **Ans: 48 km/h (not 50!)**

Q2. A particle's position is $x = 2t^3 - 3t^2 + 4$ (m). Find velocity and acceleration at $t = 2$ s.
$v = dx/dt = 6t^2 - 6t = 24 - 12 = 12$ m/s. $a = dv/dt = 12t - 6 = 18$ m/s². **Ans: 12 m/s, 18 m/s²**

Q3. A body starts from rest with 2 m/s². Distance covered in the 5th second?
$S_5 = 0 + (2/2)(2\times5 - 1) = 9$ m. **Ans: 9 m**

Q4. A car at 72 km/h brakes and stops with retardation 4 m/s². Stopping distance?
$u = 72 \times 5/18 = 20$ m/s. $s = u^2/2a = 400/8 = 50$ m. **Ans: 50 m**

Q5. A ball is thrown straight up at 20 m/s. Maximum height and time to return?
$H = u^2/2g = 400/20 = 20$ m. $T = 2u/g = 4$ s. **Ans: 20 m, 4 s**

Q6. A stone is dropped from a 45 m tall building. Time to reach the ground and final speed?
$t = \sqrt{(2H/g)} = \sqrt{9} = 3$ s. $v = gt = 30$ m/s. **Ans: 3 s, 30 m/s**

Q7. A ball is thrown up at 20 m/s from the top of a 25 m tower. When does it hit the ground?
Up +: $-25 = 20t - 5t^2 \Rightarrow t^2 - 4t - 5 = 0 \Rightarrow t = 5$ s (reject -1). **Ans: 5 s**

Q8. Trains 100 m and 150 m long approach each other at 40 m/s and 10 m/s. Time to cross?
Distance = 250 m, relative speed = 50 m/s $\Rightarrow$ $t = 5$ s. (Same direction: $250/30 \approx 8.3$ s.) **Ans: 5 s**

Q9. Acceleration $a = -2v$ (SI) and initial $v = 10$ m/s. How far does it go before stopping?
$v\ dv/dx = -2v \Rightarrow dv = -2\ dx \Rightarrow 0 - 10 = -2x \Rightarrow x = 5$ m. **Ans: 5 m**

13 Common mistakes

- Always taking average speed as $(v_1 + v_2)/2$. For equal distances use the harmonic mean.
- Taking $a = +g$ while going up. If up is +, $a = -g$ throughout.
- Thinking acceleration is 0 at the top. Only velocity is 0.
- Using $v = u + at$ when acceleration is changing.
- Not converting km/h to m/s ($\times$ 5/18).
- Subtracting the area below the axis when finding distance. For distance, take all areas as +.

14 Formula sheet

Velocity, acceleration	$v = dx/dt$, $a = dv/dt = v\,dv/dx$
Equations of motion	$v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$
n-th second	$S_n = u + (a/2)(2n - 1)$
Avg speed (half time / half distance)	$(v_1 + v_2)/2$ / $2v_1v_2/(v_1 + v_2)$
Mid-point velocity	$\sqrt{((u^2 + v^2)/2)}$
Stopping distance	$u^2/2a$ (+ $u \cdot t_r$ with reaction time)
Free fall from H	$t = \sqrt{(2H/g)}$, $v = \sqrt{(2gH)}$
Thrown up with u	$H = u^2/2g$, $T = 2u/g$
Ratios from rest	1:3:5 (each second), 1:4:9 (total)
Rest–max–rest	$v_{max} = \alpha\beta T/(\alpha + \beta)$
Relative velocity	$v_{AB} = v_A - v_B$

15 Practice questions

1. A body covers 20 m in the first 2 s and 40 m in the next 2 s (constant a). Find u and a.
2. A stone is dropped from 80 m. How far does it fall in the last second?
3. x = 4t - t² (m). When does the body stop, and how far has it gone by then?
4. Two balls are dropped together, one from 20 m and one from 45 m. Difference in their times to reach the ground?
5. A bus moves at 10 m/s. A scooter 100 m behind moves at 15 m/s. How long to catch the bus?

ANSWERS

1) u = 5 m/s, a = 5 m/s² · 2) 35 m (t = 4 s, $S_4 = 5 \times 7$) · 3) t = 2 s, x = 4 m · 4) 3 - 2 = 1 s · 5) 100/5 = 20 s

16 Quick revision: 5 minutes before the exam

BASICS

- Distance ≥ |displacement|.
- Avg velocity = displacement/time; avg speed = distance/time.
- Equal times: AM; equal distances: HM.
- v, a in the same direction ⇒ speed increases.

EQUATIONS

- $v = u + at$; $s = ut + \frac{1}{2}at^2$; $v^2 = u^2 + 2as$.
- $S_n = u + a(2n - 1)/2$.
- From rest: 1:3:5, total 1:4:9.
- Stopping distance $\propto u^2$.

GRAVITY

- $a = -g$ (up +) always.
- $H = u^2/2g$, $T = 2u/g$, $t_{up} = t_{down}$.
- Dropped from H: $t = \sqrt{(2H/g)}$, $v = \sqrt{(2gH)}$.
- At the top: $v = 0$, $a = g$.

GRAPHS

- x–t slope = v; v–t slope = a.
- v–t area = displacement.
- a–t area = Δv.
- Two x at one time: impossible.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.