FREE NOTES · NCERT + JEE/NEET LEVEL

Units & Measurement

The first step in physics: how to measure things, what a unit is, significant figures, dimensions and errors. Explained simply, with examples and solved numericals.

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01 Physical quantity & unit

Anything that **can be measured** and expressed as a number is a **physical quantity**: length, mass, time, speed, force. "Beauty" or "anger" are not physical quantities because they can't be measured against a standard.

To measure a quantity we need a **standard reference**. That standard is called a **unit**.

EVERY MEASUREMENT HAS TWO PARTS

$$\text{Physical quantity} = n \times u$$

n = numerical value, u = unit. In 5 m, n = 5 and u = metre. A bigger unit gives a smaller number: $n_1u_1 = n_2u_2$ (5 m = 500 cm).

Type	Meaning	Example
Fundamental (base) quantity	Does not depend on any other quantity	Length, mass, time
Derived quantity	Formed by combining base quantities	Speed = length/time, force = mass × acceleration

02 SI system: 7 base units

So that the whole world uses the same units, the **SI (Système Internationale)** system has been used since 1971. It has 7 base units:

Base quantity	SI unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

MEMORY TRICK

Remember the order **L M T I K N J** (Length, Mass, Time, current I, Kelvin, mole N, candela J). The same letters are used in dimensions.

Supplementary units: radian (rad) for plane angle and steradian (sr) for solid angle. Both are dimensionless.

PLANE ANGLE

$$d\theta = ds / r$$

Arc length ÷ radius. Full circle = 2π rad.

SOLID ANGLE

$$d\Omega = dA / r^2$$

Area on sphere ÷ radius². Full sphere = 4π sr.

RULES FOR WRITING UNITS

- Units named after people: name in lowercase, symbol in capital: newton (N), kelvin (K).
- Symbols have no plural: 10 kg, not "10 kgs".
- No full stop after a symbol (in mid-sentence).

03 Very large & very small lengths

Very large distances (to stars) and very small ones (atoms) need special units and methods.

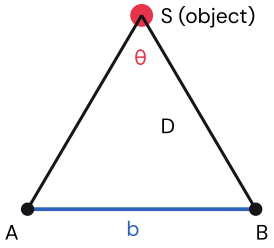
Unit	Value	Used for
Fermi (fm)	10^{-15} m	Size of a nucleus
Angstrom (Å)	10^{-10} m	Size of an atom, wavelength
Astronomical unit (AU)	1.496×10^{11} m	Average Earth–Sun distance
Light year (ly)	9.46×10^{15} m	Distance light travels in a year
Parsec (pc)	3.08×10^{16} m ≈ 3.26 ly	Largest unit of distance

Parallax method (distant stars, mountains)

Observe one object from two points A and B. Measure the distance between them, the **basis b**, and the angle formed at the object, the **parallax angle θ**.

$$D = b / \theta$$

θ must be in radians. Degrees to radians: × π/180.



Small basis b, large distance D ⇒ very small θ

Mass: for tiny particles we use the **unified atomic mass unit**: 1 u = 1/12 × (mass of a carbon-12 atom) = 1.66×10^{-27} kg.

04 Significant figures

The **reliable** digits of a measurement plus the first **uncertain** digit are its significant figures. More significant figures means a more precise measurement.

Rule	Example	Sig. figs
All non-zero digits are significant	2.345	4
Zeros between two non-zero digits are significant	2005	4
Zeros after the decimal but before the first non-zero digit are NOT significant	0.0042	2
Trailing zeros after a decimal point are significant	4.200	4
Trailing zeros without a decimal are not significant (unless measured)	4200	2
In scientific notation ($a \times 10^b$) count only the digits of a	4.20×10^3	3

CHANGING UNITS DOESN'T CHANGE SIG. FIGS

2.30 m = 230 cm = 2.30×10^2 cm = 0.00230 km. All have **3** significant figures. Use scientific notation to avoid confusion.

Significant figures in calculations

MULTIPLICATION / DIVISION

Keep as many **significant figures** as the number with the fewest sig. figs.

$4.237 \times 2.5 = 10.5925 \rightarrow \mathbf{11}$ (2 sig. figs)

ADDITION / SUBTRACTION

Keep as many **decimal places** as the number with the fewest decimal places.

$436.32 + 227.2 + 0.301 = 663.821 \rightarrow \mathbf{663.8}$

05 Rounding off

- Digit to drop is **less than 5**: previous digit unchanged. 2.743 → 2.74
- **More than 5**: previous digit +1. 2.746 → 2.75
- **Exactly 5** (nothing after it): if the previous digit is **even**, keep it; if **odd**, add 1. 2.745 → 2.74, 2.735 → 2.74

EXACT NUMBERS

Pure numbers in formulas (the 2 in r = d/2 or 2πr) have infinite significant figures. They never reduce the sig. figs of the answer.

06 Dimensions

Writing a derived quantity as **powers** of base quantities gives its **dimensional formula**. In mechanics, three brackets are enough: [M], [L], [T].

EXAMPLE: FORCE

$$F = m\,a = kg \times m/s^2 \Rightarrow [M^1\,L^1\,T^{-2}]$$

Quantity	Formula	Dimensions	SI unit
Velocity	d/t	$[M^0\,L\,T^{-1}]$	m/s
Acceleration	v/t	$[M^0\,L\,T^{-2}]$	m/s²
Force	ma	$[M\,L\,T^{-2}]$	N
Work, energy	F × d	$[M\,L^2\,T^{-2}]$	J
Power	W/t	$[M\,L^2\,T^{-3}]$	W
Momentum, impulse	mv, FΔt	$[M\,L\,T^{-1}]$	kg·m/s
Pressure, stress	F/A	$[M\,L^{-1}\,T^{-2}]$	Pa
Density	m/V	$[M\,L^{-3}]$	kg/m³
Gravitational constant G	$F\,r^2/(m_1m_2)$	$[M^{-1}\,L^3\,T^{-2}]$	N·m²/kg²
Planck's constant h	E/v	$[M\,L^2\,T^{-1}]$	J·s
Strain, angle, refractive index	ratio	dimensionless	no unit

Electric, magnetic and thermal quantities (often asked in JEE/NEET)

Quantity	Dimensions	Quantity	Dimensions
Charge q = It	$[AT]$	Potential V = W/q	$[ML^2T^{-3}A^{-1}]$
Resistance R = V/I	$[ML^2T^{-3}A^{-2}]$	Capacitance C = q/V	$[M^{-1}L^{-2}T^4A^2]$
Electric field E = F/q	$[MLT^{-3}A^{-1}]$	Magnetic field B = F/(IL)	$[MT^{-2}A^{-1}]$
Inductance L	$[ML^2T^{-2}A^{-2}]$	Magnetic flux Φ = BA	$[ML^2T^{-2}A^{-1}]$
Permittivity ε_0	$[M^{-1}L^{-3}T^4A^2]$	Permeability μ_0	$[MLT^{-2}A^{-2}]$
Specific heat c = Q/(mΔT)	$[L^2T^{-2}K^{-1}]$	Latent heat L = Q/m	$[L^2T^{-2}]$
Boltzmann constant k	$[ML^2T^{-2}K^{-1}]$	Viscosity η	$[ML^{-1}T^{-1}]$
Surface tension, spring constant	$[MT^{-2}]$	Angular velocity, frequency	$[T^{-1}]$

THESE RATIOS HAVE DIMENSIONS OF TIME [T]

L/R, RC, $\sqrt{(LC)}$: all three are measured in seconds. And $1/\sqrt{(\mu_0\varepsilon_0)}$ = speed of light, so its dimensions are $[LT^{-1}]$.

How units and dimensions relate

Statement	True / false
A quantity with no unit has no dimensions	**True**
A dimensionless quantity has no unit	**False**: angle is dimensionless but has the unit radian
Two quantities with the same dimensions must be the same quantity	**False**: work and torque are both $[ML^2T^{-2}]$ but are different things
Two quantities with different dimensions can be the same quantity	**False**

Category	Example
Dimensionless constant	π, e, pure numbers
Dimensional constant	G, h, c, ε_0
Dimensionless variable	Angle, strain, refractive index, μ (friction)
Dimensional variable	Velocity, force, length

SHORTCUT

Same dimensions: work = energy = torque $[ML^2T^{-2}]$; pressure = stress = energy density $[ML^{-1}T^{-2}]$; momentum = impulse $[MLT^{-1}]$; Planck's constant = angular momentum $[ML^2T^{-1}]$.

07 3 uses of dimensional analysis

(1) Checking whether an equation is correct

Principle of homogeneity: every term of an equation must have the same dimensions. In $s = ut + \frac{1}{2}at^2$: s → [L], ut → $[LT^{-1}][T] = [L]$, at^2 → $[LT^{-2}][T^2] = [L]$. All the same, so the equation is dimensionally correct.

NOTE

A dimensionally correct equation is not necessarily fully correct. Numbers like ½ can't be checked by dimensions. But a dimensionally wrong equation is definitely wrong.

(2) Converting from one system of units to another

$$n_2 = n_1 \, [M_1/M_2]^a \, [L_1/L_2]^b \, [T_1/T_2]^c$$

Example: 1 J in erg (CGS). $[ML^2T^{-2}]$: $n_2 = 1 \times (1000\text{ g}/1\text{ g})^1 \times (100\text{ cm}/1\text{ cm})^2 = 10^3 \times 10^4 = \mathbf{10^7\text{ erg}}$.

(3) Deriving a formula (finding a relation)

Example: a simple pendulum's time period T may depend on length l, mass m and g. Let $T = k\, m^a\, l^b\, g^c$.

$[T^1] = [M]^a\,[L]^b\,[LT^{-2}]^c \Rightarrow$ M: $a = 0$; L: $b + c = 0$; T: $-2c = 1 \Rightarrow c = -\frac{1}{2}, b = \frac{1}{2}$

$$T = k\,\sqrt{(l\,/\,g)}$$

Experiment gives $k = 2\pi$. It does not depend on mass!

LIMITATIONS

- Cannot find the value of a constant (like 2π).
- Cannot derive formulas with sin, log or e^x.
- Cannot derive a formula that depends on more than 3 quantities (using only M, L, T).
- Cannot derive a formula containing + or − terms.

08 Errors in measurement (JEE/NEET)

Every measurement has some error. **Accuracy** = how close to the true value; **precision** = how finely it was measured (least count) and how consistent repeated readings are.

EXAMPLE: ACCURACY VS PRECISION

True length is 3.678 cm. One student measures 3.5 cm (precision 0.1 cm), another 3.38 cm (precision 0.01 cm). The first is more **accurate** (closer to the true value); the second is more **precise** (finer measurement) but less accurate.

Type of error	Cause	How to reduce
Systematic	Faulty instrument (zero error), wrong method, personal habit	Correct the instrument, use the right method
Random	Sudden changes (temperature, voltage)	Take many readings and average
Least count error	Limit of the instrument's smallest reading	Use a more precise instrument

FORMULAS

Mean value: $a_{mean} = (a_1 + a_2 + \ldots + a_n)/n$

Absolute error: $\Delta a_i = |a_{mean} - a_i|$, mean absolute error $\Delta a_{mean} = \Sigma|\Delta a_i|/n$

Relative error $= \Delta a_{mean}/a_{mean}$, percentage error $=$ relative error $\times 100\%$

Combination of errors

Operation	Error
Z = A + B or Z = A − B	$\Delta Z = \Delta A + \Delta B$ (absolute errors always add)
Z = A × B or Z = A / B	$\Delta Z/Z = \Delta A/A + \Delta B/B$ (relative errors add)
$Z = A^n$	$\Delta Z/Z = n\,(\Delta A/A)$
$Z = A^p B^q / C^r$	$\Delta Z/Z = p\,\Delta A/A + q\,\Delta B/B + r\,\Delta C/C$

REMEMBER

Errors **add** even in subtraction. The quantity with the highest power needs the most accurate measurement.

09 JEE Extra + Tricks

JEE LEVEL

- Error rules: in $Z = A \cdot B$ or A/B, relative errors **add**: $\Delta Z/Z = \Delta A/A + \Delta B/B$. For $Z = A^n$, $\Delta Z/Z = n \cdot \Delta A/A$. In sums/differences, **absolute** errors add.
- Vernier LC = 1 MSD − 1 VSD; screw gauge LC = pitch / number of circular divisions. Zero error: reading − (zero error with sign).
- Same dimensions trick: Planck's constant h = angular momentum $[ML^2T^{-1}]$; work = torque = energy $[ML^2T^{-2}]$; pressure = stress = energy density $[ML^{-1}T^{-2}]$.
- Dimensional analysis cannot find dimensionless constants (2π, ½) or handle sums of terms; it fails for exponential/trig functions.
- Significant figures: in ×/÷ keep the least number of sig figs; in +/− keep the least number of decimal places.

10 Solved numericals

Q1. State the number of significant figures: (a) 0.007 (b) 2.64×10^{24} (c) 0.2370 (d) 6.320 (e) 6.032
(a) 1 (b) 3 (c) 4 (d) 4 (e) 4. **Leading zeros are never significant; trailing zeros after a decimal always are.**

Q2. A rectangle is 4.234 m long and 1.005 m wide. Find its area to the correct significant figures.
4.234 × 1.005 = 4.255170 m². Both have 4 sig. figs ⇒ **Ans: 4.255 m²**

Q3. The Moon is observed from two diametrically opposite points on Earth, basis $b = 1.276 \times 10^7$ m, parallax angle θ = 1° 54′. Find the Moon's distance.
θ = 1° 54′ = 114′ = 6840″. $1'' = 4.85 \times 10^{-6}$ rad ⇒ $\theta = 3.32 \times 10^{-2}$ rad. $D = b/\theta = 1.276 \times 10^7 / 3.32 \times 10^{-2} \approx 3.84 \times 10^8$ m. **Ans: $\approx 3.84 \times 10^8$ m**

Q4. Check whether ½mv² = mgh is dimensionally correct.
LHS: $[M][LT^{-1}]^2 = [ML^2T^{-2}]$. RHS: $[M][LT^{-2}][L] = [ML^2T^{-2}]$. Same. **Ans: Yes, dimensionally correct**

Q5. Convert 1 N into dyne (CGS: g, cm, s).
$[MLT^{-2}]$: $n_2 = 1 \times (1000)^1 \times (100)^1 \times (1)^{-2} = 10^5$. **Ans: 1 N = 10^5 dyne**

Q6. For a body in circular motion, the force $F \propto m^a v^b r^c$. Find a, b, c.
$[MLT^{-2}] = [M]^a[LT^{-1}]^b[L]^c$ ⇒ a = 1; T: −b = −2 ⇒ b = 2; L: b + c = 1 ⇒ c = −1. **Ans: $F = k\,mv^2/r$**

Q7. Five readings of a pendulum's period: 2.63, 2.56, 2.42, 2.71, 2.80 s. Find the mean, mean absolute error and % error.
Mean = 13.12/5 = 2.624 ≈ 2.62 s. Errors: 0.01, 0.06, 0.20, 0.09, 0.18 ⇒ mean error = 0.54/5 = 0.108 ≈ 0.11 s. % error = 0.11/2.62 × 100 ≈ 4%. **Ans: (2.62 ± 0.11) s, ≈ 4%**

Q8. Z = A^4 B^(1/3) / (C D^(3/2)). Relative errors in A, B, C, D are 1%, 2%, 3%, 4%. Find the % error in Z.
ΔZ/Z = 4(1) + (1/3)(2) + 1(3) + (3/2)(4) = 4 + 0.67 + 3 + 6 = 13.67%. **Ans: ≈ 13.7%**

11 Common mistakes

- Counting 4 sig. figs in 4200. Trailing zeros without a decimal are not significant (it has 2).
- Using the sig. fig rule for addition. For addition/subtraction, look at **decimal places**.
- Putting the parallax angle in degrees. Convert to radians first.
- Subtracting errors in A − B. Errors always **add**.
- Assuming "dimensionally correct" means the equation is definitely correct.
- Writing plural unit symbols (kgs) or the wrong case (Kg, Newton).

12 Formula sheet

Quantity	$Q = n\,u,\ n_1u_1 = n_2u_2$
Plane angle / solid angle	$\theta = s/r,\ \Omega = A/r^2$
Parallax distance	$D = b/\theta$ (θ in rad)
1 light year / 1 parsec	9.46×10^{15} m / 3.08×10^{16} m
1 AU / 1 Å / 1 fermi	1.496×10^{11} m / 10^{-10} m / 10^{-15} m
1 u (amu)	1.66×10^{-27} kg
Unit conversion	$n_2 = n_1\,(M_1/M_2)^a\,(L_1/L_2)^b\,(T_1/T_2)^c$
Mean absolute error	$\Delta a = \Sigma\lvert a_{mean} - a_i\rvert\ /\ n$
Sum / difference	$\Delta Z = \Delta A + \Delta B$
Product / quotient	$\Delta Z/Z = \Delta A/A + \Delta B/B$
Power	$\Delta Z/Z = n\,\Delta A/A$

13 Practice questions

1. How many significant figures are in 0.05080?
2. Find $3.0 \times 10^2 + 2.23 \times 10^2$ to the correct significant figures.
3. Find the dimensions of surface tension (force/length).
4. If the unit of length is doubled and the unit of time tripled (mass unit unchanged), what is 1 newton in the new unit?
5. Kinetic energy $E = \frac{1}{2}mv^2$. If m has a 2% error and v a 3% error, what is the % error in E?

ANSWERS

1) 4 · 2) 5.2×10^2 · 3) $[M\,T^{-2}]$ · 4) $n_2 = 1 \times (1/2)^1 \times (3)^2 = 4.5$ · 5) $2 + 2{\times}3 = 8\%$

14 Quick revision: 5 minutes before the exam

UNITS

- 7 base units: m, kg, s, A, K, mol, cd.
- Supplementary: rad, sr (dimensionless).
- Q = nu; bigger unit ⇒ smaller number.
- D = b/θ; θ in radians.
- 1 pc > 1 ly > 1 AU.

SIGNIFICANT FIGURES

- Leading zeros: no; zeros in between: yes.
- Trailing zeros after a decimal: yes.
- × ÷ : fewest sig. figs.
- + − : fewest decimal places.
- Exactly 5: even stays, odd +1.

DIMENSIONS

- F $[MLT^{-2}]$, E $[ML^2T^{-2}]$, P $[ML^2T^{-3}]$.
- Pressure $[ML^{-1}T^{-2}]$, G $[M^{-1}L^3T^{-2}]$, h $[ML^2T^{-1}]$.
- Every term has the same dimensions (homogeneity).
- Can't find constants, sin/log, or formulas with + −.

ERRORS

- + or −: absolute errors add.
- × or ÷: relative errors add.
- Power n: error × n.
- Random error: average many readings.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.