FREE NOTES · NCERT + JEE LEVEL

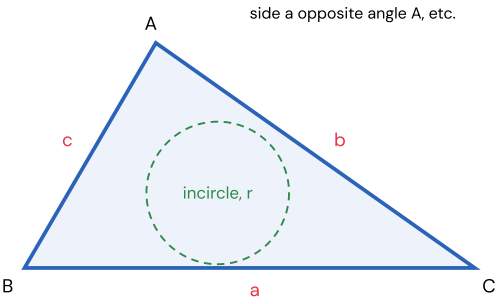
Properties of Triangles

JEE extra chapter: sine rule, cosine rule, projection formula, half-angle formulas, triangle ka area, circumradius, inradius, exradii aur triangle solve karna.

01 Sine aur cosine rules
02 Projection aur half-angle formulas
03 Area aur radii
04 Equilateral triangle shortcuts aur triangle solve karna
05 Solved examples
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07 Practice (JEE style)
08 Quick revision

01 Sine aur cosine rules

- **Sine rule**: a/sinA = b/sinB = c/sinC = 2R (R = circumradius).
- **Cosine rule**: cosA = (b² + c² − a²)/(2bc); a² = b² + c² − 2bc cosA.
- Sabse badi side sabse bade angle ke saamne; A + B + C = π.
- a² + b² = c² ⇒ C = 90°; > ⇒ acute; < ⇒ obtuse.



Notation

02 Projection aur half-angle formulas

- **Projection**: a = b cosC + c cosB (aur cyclic).
- s = (a + b + c)/2. sin(A/2) = √((s − b)(s − c)/(bc)); cos(A/2) = √(s(s − a)/(bc)); tan(A/2) = √((s − b)(s − c)/(s(s − a))).
- **Napier's analogy**: tan((B − C)/2) = ((b − c)/(b + c)) cot(A/2).
- BC ko m : n me baantne wali cevian ke liye m–n theorem (cot rule).

03 Area aur radii

Quantity	Formula
Area Δ	½bc sinA = √(s(s − a)(s − b)(s − c)) (Heron) = abc/(4R)
Circumradius R	a/(2 sinA) = abc/(4Δ)
Inradius r	Δ/s = (s − a) tan(A/2) = 4R sin(A/2)sin(B/2)sin(C/2)
Exradius r_1	Δ/(s − a) = s tan(A/2)
Upyogi	$r_1 + r_2 + r_3 - r = 4R$; circumcentre aur incentre ki doori √($R^2 - 2Rr$)

04 Equilateral triangle shortcuts aur triangle solve karna

- Equilateral (side a): $\Delta = (\sqrt{3}/4)a^2$, $R = a/\sqrt{3}$, $r = a/(2\sqrt{3})$, $R = 2r$, $r_1 = r_2 = r_3 = (\sqrt{3}/2)a$.
- Ambiguous case (a, b, A diye): $a < b \sin A \Rightarrow$ triangle nahi; $a = b \sin A \Rightarrow$ ek (right); $b \sin A < a < b \Rightarrow$ do; $a \geq b \Rightarrow$ ek.
- Trick: har triangle me $R \geq 2r$ (Euler); barabar sirf equilateral me.

05 Solved examples

Q1. △ABC me a = 3, b = 5, c = 7. Sabse bada angle?

cos C = (9 + 25 − 49)/30 = −½ ⇒ C = 120°.

Q2. 13, 14, 15 sides wale triangle ka area aur inradius?

s = 21, Δ = √(21·8·7·6) = 84; r = Δ/s = 4.

Q3. Usi triangle ka R?

R = abc/(4Δ) = 2730/336 = 8.125.

06 Common mistakes

- Ek formula me degrees aur radians mix karna.
- Sine rule me ambiguous case bhoolna.
- r = Δ/(s − a) likhna (wo r_1 hai).
- s ko perimeter lena (semi-perimeter hai).

07 Practice (JEE style)

1. 6 side ke equilateral ka circumradius?
2. a cosA = b cosB ho to triangle?
3. b = 4, c = 5, A = 30° pe area?
4. 3-4-5 triangle ka inradius?
5. $r_1 + r_2 + r_3 - r$ = ?

ANSWERS

1) 2√3 · 2) Isosceles or right-angled · 3) 5 · 4) 1 · 5) 4R

08 Quick revision: exam se 5 minute pehle

RULES

- a/sinA = 2R
- cosA = $(b^2+c^2-a^2)/2bc$
- a = b cosC + c cosB

AREA

- ½bc sinA
- Heron
- abc/4R

RADII

- $r = \Delta/s$
- $r_1 = \Delta/(s-a)$
- $R \geq 2r$

HALF ANGLES

- $\tan(A/2)$ formula
- Napier's analogy

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.