FREE NOTES · NCERT + JEE LEVEL

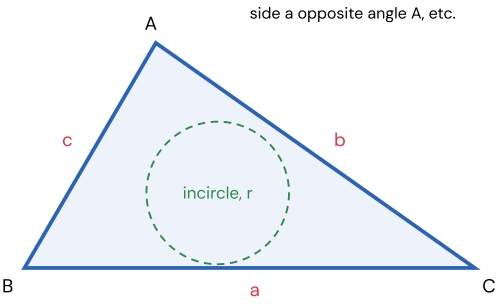
Properties of Triangles

JEE extra chapter: sine rule, cosine rule, projection formula, half-angle formulas, area of a triangle, circumradius, inradius, exradii and solution of triangles.

01 Sine and cosine rules
02 Projection and half-angle formulas
03 Area and radii
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08 Quick revision

01 Sine and cosine rules

- **Sine rule**: a/sinA = b/sinB = c/sinC = 2R (R = circumradius).
- **Cosine rule**: cosA = (b² + c² − a²)/(2bc); a² = b² + c² − 2bc cosA.
- Largest side is opposite the largest angle; A + B + C = π.
- If a² + b² = c² ⇒ C = 90°; > ⇒ acute; < ⇒ obtuse.



Notation

02 Projection and half-angle formulas

- **Projection**: $a = b\cos C + c\cos B$ (and cyclic).
- $s = (a + b + c)/2$. $\sin(A/2) = \sqrt{((s - b)(s - c)/(bc))}$; $\cos(A/2) = \sqrt{(s(s - a)/(bc))}$; $\tan(A/2) = \sqrt{((s - b)(s - c)/(s(s - a)))}$.
- **Napier's analogy**: $\tan((B - C)/2) = ((b - c)/(b + c))\cot(A/2)$.
- m–n theorem (cot rule) for a cevian dividing BC in m : n.

03 Area and radii

Quantity	Formula
Area Δ	$\frac{1}{2}bc\sin A = \sqrt{(s(s - a)(s - b)(s - c))}$ (Heron) $= abc/(4R)$
Circumradius R	$a/(2\sin A) = abc/(4\Delta)$
Inradius r	$\Delta/s = (s - a)\tan(A/2) = 4R\sin(A/2)\sin(B/2)\sin(C/2)$
Exradius r_1	$\Delta/(s - a) = s\tan(A/2)$
Useful	$r_1 + r_2 + r_3 - r = 4R$; distance between circumcentre and incentre $\sqrt{(R^2 - 2Rr)}$

04 Equilateral triangle shortcuts and solving triangles

- Equilateral (side a): $\Delta = (\sqrt{3}/4)a^2$, $R = a/\sqrt{3}$, $r = a/(2\sqrt{3})$, $R = 2r$, $r_1 = r_2 = r_3 = (\sqrt{3}/2)a$.
- Ambiguous case (given a, b, A): if $a < b \sin A$ no triangle; $a = b \sin A$ one (right); $b \sin A < a < b$ two triangles; $a \geq b$ one.
- Trick: in any triangle, $R \geq 2r$ (Euler); equality only for equilateral.

05 Solved examples

Q1. In $\triangle ABC$, $a = 3$, $b = 5$, $c = 7$. Find the largest angle.

$\cos C = (9 + 25 - 49)/(2 \cdot 3 \cdot 5) = -15/30 = -\frac{1}{2} \Rightarrow C = 120°$.

Q2. Find the area and inradius of a triangle with sides 13, 14, 15.

$s = 21$, $\Delta = \sqrt{(21 \cdot 8 \cdot 7 \cdot 6)} = 84$; $r = \Delta/s = 4$.

Q3. For the same triangle find R.

$R = abc/(4\Delta) = 2730/336 = 8.125$.

06 Common mistakes

- Using degrees and radians mixed in the same formula.
- Forgetting the ambiguous case in sine rule problems.
- Writing $r = \Delta/(s - a)$ (that is r_1).
- Taking s as perimeter instead of semi-perimeter.

07 Practice (JEE style)

1. Circumradius of equilateral triangle of side 6?
2. If $a \cos A = b \cos B$, the triangle is?
3. Area with $b = 4$, $c = 5$, $A = 30°$?
4. Inradius of 3-4-5 triangle?
5. $r_1 + r_2 + r_3 - r = ?$

ANSWERS

1) $2\sqrt{3}$ · 2) Isosceles or right-angled · 3) 5 · 4) 1 · 5) 4R

08 Quick revision: 5 minutes before the exam

RULES

- $a/\sin A = 2R$
- $\cos A = (b^2+c^2-a^2)/2bc$
- $a = b \cos C + c \cos B$

AREA

- $\frac{1}{2}bc \sin A$
- Heron
- $abc/4R$

RADII

- $r = \Delta/s$
- $r_1 = \Delta/(s-a)$
- $R \geq 2r$

HALF ANGLES

- $\tan(A/2)$ formula
- Napier's analogy

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.