FREE NOTES · NCERT + JEE LEVEL

Mathematical Reasoning

Extra chapter (old CBSE Class 11, asked in JEE Main earlier): statements, negation, compound statements, connectives, truth tables, implications, converse, contrapositive, tautology and contradiction.

01 Statements and negation
02 Connectives and truth tables
03 Implications and laws
04 Validating statements
05 Solved examples
06 Common mistakes
07 Practice (JEE style)
08 Quick revision

01 Statements and negation

- A **statement** is a sentence that is either true or false, not both. Questions, commands, exclamations and opinions are not statements.
- **Negation** ~p: "It is not the case that p". ~(~p) = p.
- Negation of quantifiers: ~(for all x, p(x)) = there exists x such that ~p(x); ~(there exists x, p(x)) = for all x, ~p(x).

02 Connectives and truth tables

p	q	p ∧ q	p ∨ q	p → q	p ↔ q
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

- p → q is false **only** when p is true and q is false.
- Number of rows in truth table with n statements = 2^n.

03 Implications and laws

- For p → q: **converse** q → p; **inverse** ~p → ~q; **contrapositive** ~q → ~p (equivalent to the original).
- p → q ≡ ~p ∨ q; ~(p → q) ≡ p ∧ ~q.
- **De Morgan**: ~(p ∧ q) ≡ ~p ∨ ~q; ~(p ∨ q) ≡ ~p ∧ ~q.
- p ↔ q ≡ (p → q) ∧ (q → p).
- **Tautology**: always true (p ∨ ~p); **contradiction**: always false (p ∧ ~p).

04 Validating statements

- Direct method: assume p true, show q true.
- Contrapositive method: assume ~q, show ~p.
- Contradiction: assume p and ~q, reach an impossibility.
- Counter-example disproves a "for all" statement.
- Trick for JEE MCQs: replace each statement by T/F values and test the options quickly; or convert → into ∨ and simplify.

05 Solved examples

Q1. Write the contrapositive of "If a number is divisible by 6, it is divisible by 3."

If a number is not divisible by 3, it is not divisible by 6.

Q2. Show that $(p \wedge q) \to p$ is a tautology.

$(p \wedge q) \to p \equiv \sim(p \wedge q) \vee p \equiv \sim p \vee \sim q \vee p \equiv T$.

Q3. Negate: "All students are intelligent."

Some student is not intelligent (there exists a student who is not intelligent).

06 Common mistakes

- Thinking converse is equivalent to the original (contrapositive is).
- Negating "all" as "none" (correct: "some … not").
- Treating $p \to q$ as false when p is false.
- Calling questions or commands statements.

07 Practice (JEE style)

1. $\sim(p \vee q) \equiv$?
2. $p \to q \equiv$?
3. Rows in truth table for 3 statements?
4. $p \vee \sim p$ is a?
5. Contrapositive of $p \to q$?

ANSWERS

1) $\sim p \wedge \sim q$ · 2) $\sim p \vee q$ · 3) 8 · 4) Tautology · 5) $\sim q \to \sim p$

08 Quick revision: 5 minutes before the exam

BASICS

- Statement: T or F
- $\sim(\sim p) = p$

CONNECTIVES

- $\wedge$ both true
- $\vee$ at least one
- $\to$ false only T→F

LAWS

- De Morgan
- $p \rightarrow q \equiv \sim p \vee q$
- Contrapositive equivalent

SPECIAL

- Tautology $p \vee \sim p$
- Contradiction $p \wedge \sim p$

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.