FREE NOTES · NCERT + JEE LEVEL

Probability

Random experiments, outcomes and sample space, events and their types, algebra of events, the axiomatic approach, probability of equally likely outcomes, "or", "and", "not" events and addition theorem.

01 Random experiment
02 Sample space S
03 Events & their types
04 Algebra of events
05 Axiomatic probability
06 Addition rule & complement
07 Standard experiments at a glance
08 JEE Extra + Tricks
09 Solved examples
10 Common mistakes
11 Formula sheet & practice
12 Quick revision

01 Random experiment

An experiment with more than one possible outcome where we can't predict the result in advance (tossing a coin, rolling a die). Each possible result is an **outcome**.

02 Sample space S

- One coin: {H, T}. Two coins: {HH, HT, TH, TT}. n coins: 2^n outcomes.
- One die: {1, ..., 6}. Two dice: 36 ordered pairs.
- Card pack: 52 cards, 4 suits (♠ ♣ black, ♥ ♦ red) of 13; face cards J, Q, K = 12.

	1	2	3	4	5	6
1	1,1	2,1	3,1	4,1	5,1	6,1
2	1,2	2,2	3,2	4,2	5,2	6,2
3	1,3	2,3	3,3	4,3	5,3	6,3
4	1,4	2,4	3,4	4,4	5,4	6,4
5	1,5	2,5	3,5	4,5	5,5	6,5
6	1,6	2,6	3,6	4,6	5,6	6,6

36 outcomes; sum 7 shaded ⇒ P = 6/36 = 1/6

Sample space of two dice

03 Events & their types

An **event** is any subset of S.

Type	Meaning
Impossible	φ (never happens)
Sure	S itself
Simple	One sample point
Compound	More than one sample point
Mutually exclusive	A ∩ B = φ (can't happen together)
Exhaustive	A ∪ B ∪ … = S

04 Algebra of events

Words	Set notation
Not A	A′ = S − A
A or B	A ∪ B
A and B	A ∩ B
A but not B	A − B = A ∩ B′
Exactly one of A, B	(A ∩ B′) ∪ (A′ ∩ B)
Neither A nor B	A′ ∩ B′ = (A ∪ B)′

05 Axiomatic probability

1. $0 \le P(E) \le 1$ for every event.
2. $P(S) = 1$.
3. If E, F are mutually exclusive, $P(E \cup F) = P(E) + P(F)$.

$$\text{Equally likely: } P(E) = n(E)/n(S)$$

06 Addition rule & complement

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually exclusive: $P(A \cup B) = P(A) + P(B)$

$P(\text{not } A) = 1 - P(A)$

$P(A \text{ but not } B) = P(A) - P(A \cap B)$

Odds in favour of A = P(A) : P(A′).

07 Standard experiments at a glance

- Two dice: sum 7 occurs 6 times (most likely sum); doublets 6 times.
- Three coins: P(at least one head) = 1 - 1/8 = 7/8.
- Cards: P(king) = 4/52 = 1/13; P(heart) = 13/52 = 1/4; P(red king) = 2/52.
- Selecting r from n: use ${}^{n}C_{r}$ for both n(E) and n(S).

08 JEE Extra + Tricks

JEE LEVEL

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; exactly one of A, B: $P(A) + P(B) - 2P(A \cap B)$.
- Odds in favour a : b $\Rightarrow$ $P = a/(a + b)$.
- Two dice: P(sum = 7) = 6/36 (most likely sum). Three dice total outcomes 216.
- Coins: P(at least one head in n tosses) = $1 - (½)^n$.
- Complement trick: "at least one" $\Rightarrow$ 1 − P(none).

09 Solved examples

Q1. Two dice are thrown. P(sum is a prime)?

Sums 2, 3, 5, 7, 11 occur 1 + 2 + 4 + 6 + 2 = 15 times $\Rightarrow$ **15/36 = 5/12**.

Q2. P(A) = 0.42, P(B) = 0.48, $P(A \cap B)$ = 0.16. Find $P(A \cup B)$ and P(not A).

0.42 + 0.48 − 0.16 = **0.74**; 1 − 0.42 = **0.58**.

Q3. 4 cards drawn from a pack. P(all four are aces)?

${}^{4}C_{4}/{}^{52}C_{4}$ = **1/270725**.

Q4. A committee of 2 from 4 men and 3 women. P(both women)?

${}^{3}C_{2}/{}^{7}C_{2}$ = 3/21 = **1/7**.

10 Common mistakes

- Counting (1, 2) and (2, 1) as one outcome for two dice (they are different).
- Adding P(A) + P(B) for events that can overlap.
- Getting a probability > 1 or negative and not noticing.
- Confusing mutually exclusive with exhaustive.

11 Formula sheet & practice

Classical	$P(E) = n(E)/n(S)$
Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Complement	$P(A') = 1 - P(A)$
Range	$0 \le P \le 1$

1. A coin is tossed 3 times. P(exactly 2 heads)?
2. A die is rolled. P(number > 4)?
3. If P(A) = 3/5, P(B) = 1/5 and A, B mutually exclusive, P(A or B)?
4. P(a leap year has 53 Sundays)?

ANSWERS

1) 3/8 · 2) 1/3 · 3) 4/5 · 4) 2/7

12 Quick revision: 5 minutes before the exam

BASICS

- Event = subset of S.
- n coins: 2^n; 2 dice: 36.

AXIOMS

- $0 \le P \le 1$, P(S) = 1.
- Exclusive ⇒ add.

RULES

- $P(A \cup B)$ formula.
- P(A′) = 1 − P(A).

COUNTING

- Use ${}^nC_r$ for selections.
- "At least one" = 1 − P(none).

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.