FREE NOTES · NCERT + JEE LEVEL

Limits & Derivatives

Limit ka idea, left aur right limits, limits ka algebra, polynomial, rational aur trigonometric functions ki standard limits, first principle se derivative, differentiation ke rules aur standard functions ke derivatives.

01 Limit ka idea
02 Limits ka algebra
03 Standard limits
04 Trigonometric limits
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01 Limit ka idea

$\lim_{x \to a} f(x) = L$ matlab x jab a ke paas jaata (par $x \neq a$), f(x) L ke jitna chahe utna paas pahunchta. f(a) exist na kare tab bhi chalega.

- **Left-hand limit** $\lim_{x \to a^-} f(x)$ aur **right-hand limit** $\lim_{x \to a^+} f(x)$.
- Limit tabhi exist karti jab LHL = RHL.
- Example: f(x) = |x|/x ka 0 pe LHL = −1, RHL = 1, isliye limit nahi.

02 Limits ka algebra

$$\lim [f \pm g] = \lim f \pm \lim g;\ \lim [f \cdot g] = \lim f \cdot \lim g$$

$$\lim [f/g] = \lim f \,/\, \lim g \text{ (agar } \lim g \neq 0);\ \lim [k\, f] = k \lim f$$

Polynomials: seedha value daalo. Rational: value daalo; 0/0 aaye to factorise karke cancel, ya rationalise.

03 Standard limits

$$\lim_{x \to a} (x^n - a^n)/(x - a) = n\, a^{n-1}$$

$\lim_{x \to 0} (e^x - 1)/x = 1;\ \lim_{x \to 0} \log(1 + x)/x = 1$ (JEE)

Example: $\lim_{x \to 1} (x^{15} - 1)/(x^{10} - 1)$ = (15/10) = 3/2 (dono ko x − 1 se bhaag do).

04 Trigonometric limits

$$\lim_{x \to 0} \sin x / x = 1$$

$\lim_{x \to 0} (1 - \cos x)/x = 0;\ \lim_{x \to 0} \tan x / x = 1$

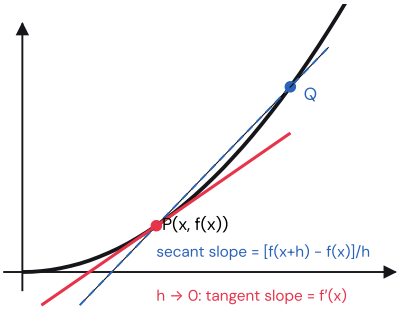
$\lim_{x \to 0} \sin ax / bx = a/b$

x radian me hona chahiye. Sandwich theorem se prove hota (cos x < sin x / x < 1).

05 Derivative: matlab kya

Derivative f′(x), f ke badalne ki **instantaneous rate** hai, yaani us point pe curve ki tangent ka slope.

Physics se jod: velocity = displacement ka derivative; acceleration = velocity ka derivative.



h → 0 pe secant tangent ban jaati

06 First principles se derivative

$$f'(x) = \lim_{h \to 0} [f(x+h) - f(x)] / h$$

Example $f(x) = x^2$: $[(x+h)^2 - x^2]/h = 2x + h \to \mathbf{2x}$. Example $f(x) = \sin x$: $[\sin(x+h) - \sin x]/h = 2\cos(x + h/2)\sin(h/2)/h \to \mathbf{\cos x}$.

07 Rules & standard derivatives

Rule	Formula
Sum	$(u \pm v)' = u' \pm v'$
Product (Leibniz)	$(uv)' = u'v + uv'$
Quotient	$(u/v)' = (u'v - uv')/v^2$
Constant multiple	$(ku)' = k\,u'$

f(x)	f′(x)	f(x)	f′(x)
x^n	$n\,x^{n-1}$	c	0
sin x	cos x	cos x	−sin x
tan x	$\sec^2 x$	cot x	$-\mathrm{cosec}^2 x$
sec x	sec x tan x	cosec x	−cosec x cot x

08 JEE Extra + Tricks

JEE LEVEL

- Standard limits: $\lim \frac{\sin x}{x} = 1$, $\lim \frac{1 - \cos x}{x^2} = \frac{1}{2}$, $\lim \frac{e^x - 1}{x} = 1$, $\lim \frac{a^x - 1}{x} = \ln a$, $\lim \frac{\ln(1 + x)}{x} = 1$, $\lim \frac{x^n - a^n}{x - a} = na^{n-1}$.
- 1^∞ form: $\lim f^g = e^{\lim g(f - 1)}$.
- L'Hôpital rule (0/0 ya ∞/∞): numerator aur denominator alag differentiate.
- 0 ke paas expansions: $\sin x \approx x - x^3/6$, $\cos x \approx 1 - x^2/2$, $\tan x \approx x + x^3/3$, $e^x \approx 1 + x + x^2/2$, $\ln(1 + x) \approx x - x^2/2$.
- $x \to \infty$ pe rational function: sabse badi powers compare karo.

09 Solved examples

Q1. $\lim_{x\to 2} (x^2 - 4)/(x - 2)$.

$(x - 2)(x + 2)/(x - 2) \to$ **4**.

Q2. $\lim_{x\to 0} \sin 4x / \sin 2x$.

$(\sin 4x/4x)\cdot 4x / [(\sin 2x/2x)\cdot 2x] \to$ **2**.

Q3. $\lim_{x\to 0} (\sqrt{(1 + x)} - 1)/x$.

Rationalise: $x/[x(\sqrt{(1+x)} + 1)] \to$ **½**.

Q4. $(x + 1)(x^2 - 3)$ ka derivative.

$(x^2 - 3) + (x + 1)(2x) =$ **$3x^2 + 2x - 3$**.

Q5. $(x + \cos x)/\tan x$ ka derivative.

$[(1 - \sin x) \tan x - (x + \cos x) \sec^2 x]/\tan^2 x$.

10 Common mistakes

- Kehna ki limit hamesha f(a) ke barabar (sirf continuous f me).
- sin x/x me degree lagana.
- Product rule (uv)′ = u′v′ likhna (galat).
- Quotient rule me numerator ulta (u′v − uv′ hai, uv′ − u′v nahi).
- cos x ka derivative sin x likhna (minus bhool gaye).

11 Formula sheet & practice

Standard	$(x^n - a^n)/(x - a) \to na^{n-1}$
Trig	$\sin x/x \to 1$
First principle	$\lim [f(x+h) - f(x)]/h$
Product / quotient	$u'v + uv'$; $(u'v - uv')/v^2$

1. $\lim_{x\to 0} (1 - \cos 2x)/x^2$.
2. x^{-3} ka derivative.
3. sin x cos x ka derivative.

4. $f(x) = x^{100} + x^{99} + \ldots + x + 1$ ho to $f'(1)$.

ANSWERS

1) 2 · 2) $-3x^{-4}$ · 3) cos 2x · 4) 5050

12 Quick revision: exam se 5 minute pehle

LIMITS

- LHL = RHL ⇒ limit exist.
- 0/0 ⇒ factorise/rationalise.

STANDARD

- $(x^n - a^n)/(x - a) \to na^{n-1}$.
- $\sin x/x \to 1$ (radian).

DERIVATIVE

- $f' = \lim [f(x+h) - f(x)]/h$.
- Tangent ka slope.

RULES

- $(uv)' = u'v + uv'$.
- $(u/v)' = (u'v - uv')/v^2$.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.