FREE NOTES · NCERT + JEE LEVEL

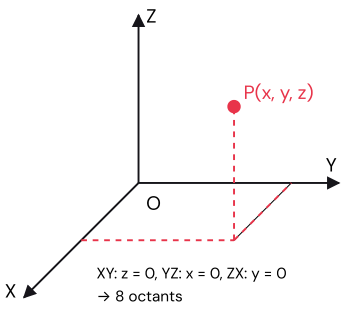
Introduction to 3D Geometry

Coordinate axes and planes in space, octants and signs, coordinates of a point, distance between two points and the section formula in three dimensions.

01 Axes & coordinate planes
02 Coordinates of a point
03 Octants & signs
04 Distance formula
05 Section formula
06 Useful results
07 Formula sheet
08 JEE Extra + Tricks
09 Solved examples
10 Common mistakes
11 Practice questions
12 Quick revision

01 Axes & coordinate planes

Three mutually perpendicular lines through O: X, Y and Z axes (right-handed system). Each pair forms a plane: **XY** (z = 0), **YZ** (x = 0), **ZX** (y = 0).



Point P(x, y, z) in space

02 Coordinates of a point

- P(x, y, z): x = distance from YZ plane, y = from ZX plane, z = from XY plane (with sign).
- On the x-axis: (x, 0, 0); y-axis: (0, y, 0); z-axis: (0, 0, z).
- In the XY plane: (x, y, 0); YZ plane: (0, y, z); ZX plane: (x, 0, z).
- Foot of perpendicular from (x, y, z) on XY plane = (x, y, 0); on x-axis = (x, 0, 0).

03 Octants & signs

Octant	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

Trick: octants I–IV have z positive, V–VIII z negative; within each group the (x, y) signs follow the 2D quadrants.

04 Distance formula

$$PQ = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2)}$$

$OP = \sqrt{(x^2 + y^2 + z^2)}$

Distance of (x, y, z) from x-axis = $\sqrt{(y^2 + z^2)}$; from XY plane = $|z|$.

05 Section formula

Internal $m : n \Rightarrow ((mx_2 + nx_1)/(m+n), (my_2 + ny_1)/(m+n), (mz_2 + nz_1)/(m+n))$

External $m : n \Rightarrow$ replace $+$ by $-$: $((mx_2 - nx_1)/(m-n), \ldots)$

Midpoint $= ((x_1+x_2)/2, (y_1+y_2)/2, (z_1+z_2)/2)$

Centroid of triangle $= ((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3, (z_1+z_2+z_3)/3)$

06 Useful results

- **Collinear** points A, B, C: AB + BC = AC.
- Ratio in which a plane divides a segment: e.g. YZ plane (x = 0) divides AB in ratio $-x_1 : x_2$ (take k : 1 and set x-coordinate = 0).
- Triangle type: compare the three side lengths (equal ⇒ equilateral; Pythagoras ⇒ right-angled).

07 Formula sheet

Distance	$\sqrt{(\Delta x^2 + \Delta y^2 + \Delta z^2)}$
Section (internal)	$(mx_2 + nx_1)/(m + n)$, ...
Midpoint	average of coordinates
Centroid	sum/3

08 JEE Extra + Tricks

JEE LEVEL

- Section formula and centroid extend directly to 3D (add z).
- Direction cosines $l^2 + m^2 + n^2 = 1$; angle between lines $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$.
- Distance of (x, y, z) from x-axis = $\sqrt{(y^2 + z^2)}$; from origin $\sqrt{(x^2 + y^2 + z^2)}$.
- Points collinear iff the ratios of differences of coordinates are equal (or area/cross product zero).

09 Solved examples

Q1. Distance between (2, -1, 3) and (-2, 1, 3).

$\sqrt{(16 + 4 + 0)} = \mathbf{2\sqrt{5}}$.

Q2. Show that (-2, 3, 5), (1, 2, 3), (7, 0, -1) are collinear.

AB = $\sqrt{14}$, BC = $2\sqrt{14}$, AC = $3\sqrt{14}$ ⇒ AB + BC = AC ✓.

Q3. Point dividing (1, -2, 3) and (3, 4, -5) in ratio 2 : 3 internally.

((6 + 3)/5, (8 - 6)/5, (-10 + 9)/5) = **(9/5, 2/5, -1/5)**.

Q4. In what ratio does the YZ plane divide the join of (-2, 4, 7) and (3, -5, 8)?

(3k - 2)/(k + 1) = 0 ⇒ k = 2/3 ⇒ **2 : 3** internally.

10 Common mistakes

- Mixing up "distance from x-axis" ($\sqrt{(y^2 + z^2)}$) and "distance from YZ plane" ($|x|$).
- Wrong octant signs for V–VIII.
- Using + in the external section formula.

11 Practice questions

1. In which octant is (−3, 1, −2)?
2. Coordinates of a point on the y-axis at distance 5 from origin.
3. Find x if the distance between (5, −1, 7) and (x, 5, 1) is 9.
4. Centroid of (1, 2, 3), (0, 4, 1), (−1, −1, −3).

ANSWERS

1) VI · 2) (0, ±5, 0) · 3) x = 2 or 8 · 4) (0, 5/3, 1/3)

12 Quick revision: 5 minutes before the exam

BASICS

- 3 axes, 3 planes, 8 octants.
- XY: z = 0.

DISTANCE

- $\sqrt{(\Delta x^2+\Delta y^2+\Delta z^2)}$.
- From x-axis $\sqrt{(y^2+z^2)}$.

SECTION

- Same as 2D + z.
- Plane ratio: set coordinate 0.

COLLINEAR

- AB + BC = AC.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.