FREE NOTES · NCERT + JEE LEVEL

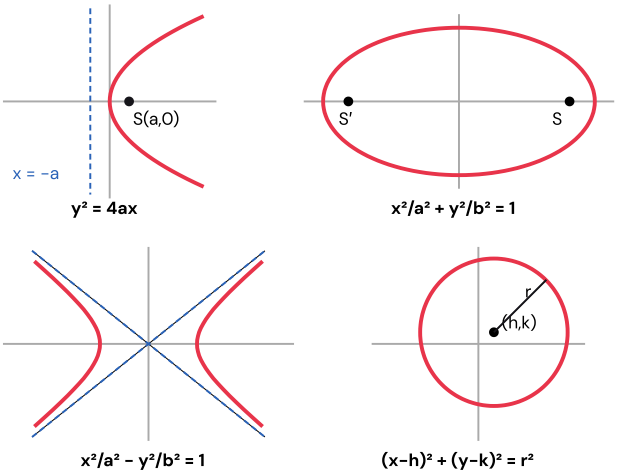
Conic Sections

Cone ke sections, circle, parabola, ellipse aur hyperbola: standard equations, focus, directrix, vertices, axes, eccentricity aur latus rectum, aur wo comparison table jo har board paper me kaam aati.

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01 Cone ke sections

Double cone ko plane se kaato. β = plane ka axis se angle, α = semi-vertical angle: β = 90° ⇒ **circle**; α < β < 90° ⇒ **ellipse**; β = α ⇒ **parabola**; 0 ≤ β < α ⇒ **hyperbola**. Vertex se kaato to degenerate cases (point, line, lines ka joda).



Standard parabola, ellipse, hyperbola aur circle

02 Circle

$$(x-h)^2+(y-k)^2=r^2$$

General: $x^2+y^2+2gx+2fy+c=0 \Rightarrow$ centre $(-g,-f)$, $r=\sqrt{(g^2+f^2-c)}$

Centre se fixed doori r wale points ka set. $g^2+f^2-c=0$ ho to point circle; <0 ho to koi real circle nahi.

03 Parabola

Wo points jo ek fixed point (**focus**) aur ek fixed line (**directrix**) se barabar doori pe hon. Eccentricity e = 1.

Equation	Focus	Directrix	Axis	Khulta
$y^2 = 4ax$	(a, 0)	x = -a	x-axis	Right
$y^2 = -4ax$	(-a, 0)	x = a	x-axis	Left
$x^2 = 4ay$	(0, a)	y = -a	y-axis	Upar
$x^2 = -4ay$	(0, -a)	y = a	y-axis	Neeche

Latus rectum = 4a (focus se guzarti, axis ke $\perp$ chord). Vertex origin pe.

04 Ellipse

Do foci se dooriyon ka jod constant (= 2a).

$$x^2/a^2 + y^2/b^2 = 1 \ (a > b)$$

$c^2 = a^2 - b^2$, $e = c/a < 1$

Foci (±c, 0), vertices (±a, 0), major axis 2a, minor axis 2b, latus rectum $2b^2/a$. Bada denominator y^2 ke neeche ho ($x^2/b^2 + y^2/a^2 = 1$) to major axis y pe: foci (0, ±c). e → 0 pe circle.

05 Hyperbola

Do foci se dooriyon ka fark constant (= 2a).

$$x^2/a^2 - y^2/b^2 = 1$$

$c^2 = a^2 + b^2$, $e = c/a > 1$

Foci (±c, 0), vertices (±a, 0), transverse axis 2a, conjugate axis 2b, latus rectum $2b^2/a$. $y^2/a^2 - x^2/b^2 = 1$ me transverse axis y pe. a = b ho to rectangular hyperbola (e = √2).

06 Comparison table

	Parabola $y^2 = 4ax$	Ellipse $x^2/a^2 + y^2/b^2 = 1$	Hyperbola $x^2/a^2 - y^2/b^2 = 1$
e	1	< 1, $\sqrt{(1 - b^2/a^2)}$	> 1, $\sqrt{(1 + b^2/a^2)}$
c	—	$\sqrt{(a^2 - b^2)}$	$\sqrt{(a^2 + b^2)}$
Foci	(a, 0)	(±c, 0)	(±c, 0)
Vertices	(0, 0)	(±a, 0)	(±a, 0)
Latus rectum	4a	$2b^2/a$	$2b^2/a$
Khaas property	SP = PM	SP + S'P = 2a	|SP − S'P| = 2a

07 Conic jaldi kaise pehchanein

- Sirf ek variable ka square ⇒ **parabola**. Jiska square wo axis ke perpendicular (y^2 ⇒ axis x pe).
- Dono ke square, dono +, alag denominators ⇒ **ellipse**; bada denominator major axis batata.
- Dono ke square, ek minus ⇒ **hyperbola**; positive wala term transverse axis batata.
- Dono ke square, same coefficient, + ⇒ **circle**.

08 JEE Extra + Tricks

JEE LEVEL

- Tangent conditions: $y = mx + c$ parabola $y^2 = 4ax$ ko chhoota agar $c = a/m$; ellipse $c^2 = a^2m^2 + b^2$; hyperbola $c^2 = a^2m^2 - b^2$.
- Parametric points: parabola $(at^2, 2at)$; ellipse $(a\cos\theta, b\sin\theta)$; hyperbola $(a\sec\theta, b\tan\theta)$.
- Latus rectum: parabola $4a$; ellipse aur hyperbola $2b^2/a$. Eccentricity: ellipse $e^2 = 1 - b^2/a^2$, hyperbola $e^2 = 1 + b^2/a^2$.
- Parabola ki focal chord: $t_1t_2 = -1$; uske siron ki tangents perpendicular aur directrix pe milti.
- Director circle: ellipse $x^2 + y^2 = a^2 + b^2$; jahan se perpendicular tangents.
- Circle: (x_1, y_1) se tangent ki lambai = $\sqrt{S_1}$; do circles orthogonal agar $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

09 Solved examples

Q1. $x^2 + y^2 - 4x - 8y - 45 = 0$ ka centre aur radius.

$g = -2$, $f = -4 \Rightarrow$ centre $(2, 4)$, $r = \sqrt{(4 + 16 + 45)} = \mathbf{\sqrt{65}}$.

Q2. $y^2 = -8x$ ka focus, directrix, latus rectum.

$4a = 8$, $a = 2$, left khulta $\Rightarrow$ focus $(-2, 0)$, directrix $x = 2$, LR = 8.

Q3. $x^2/25 + y^2/9 = 1$ ke foci aur e.

$a = 5$, $b = 3$, $c = 4 \Rightarrow$ foci $(\pm4, 0)$, $e = \mathbf{4/5}$.

Q4. $9y^2 - 4x^2 = 36$ ke foci aur e.

$y^2/4 - x^2/9 = 1 \Rightarrow a = 2$, $b = 3$, $c = \sqrt{13} \Rightarrow$ foci $(0, \pm\sqrt{13})$, $e = \mathbf{\sqrt{13}/2}$.

10 Common mistakes

- Hyperbola me $c^2 = a^2 - b^2$ lagana ($a^2 + b^2$ hai).
- Bada number y^2 ke neeche ho phir bhi x^2 wale ko a lena.
- Parabola ka latus rectum 2a lena (4a hai).
- a aur b padhne se pehle RHS ko 1 na banana.

11 Practice questions

1. Centre (−3, 2) aur radius 4 wale circle ki equation.
2. Focus (0, −3) aur vertex origin wale parabola ki equation.
3. Vertices (±13, 0) aur foci (±5, 0) wala ellipse.
4. Foci (±5, 0) aur transverse axis 8 wala hyperbola.

ANSWERS

1) $x^2 + y^2 + 6x - 4y - 3 = 0$ · 2) $x^2 = -12y$ · 3) $x^2/169 + y^2/144 = 1$ · 4) $x^2/16 - y^2/9 = 1$

12 Quick revision: exam se 5 minute pehle

CIRCLE

- Centre $(-g, -f)$.
- $r = \sqrt{(g^2 + f^2 - c)}$.

PARABOLA

- $y^2 = 4ax$: $S(a,0)$, $x = -a$.
- LR = 4a, e = 1.

ELLIPSE

- $c^2 = a^2 - b^2$, $e < 1$.
- LR = $2b^2/a$.

HYPERBOLA

- $c^2 = a^2 + b^2$, $e > 1$.
- LR = $2b^2/a$.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.