FREE NOTES · NCERT + JEE LEVEL

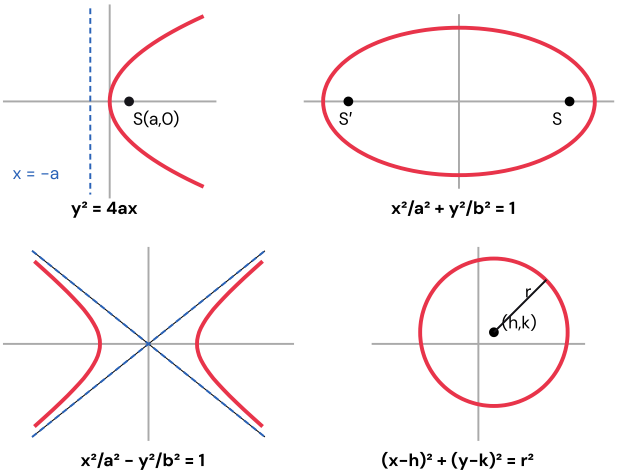
Conic Sections

Sections of a cone, circle, parabola, ellipse and hyperbola: standard equations, focus, directrix, vertices, axes, eccentricity and latus rectum, with the comparison table every board paper needs.

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01 Sections of a cone

Cut a double cone by a plane. If β = angle of the plane with the axis and α = semi-vertical angle: β = 90° ⇒ **circle**; α < β < 90° ⇒ **ellipse**; β = α ⇒ **parabola**; 0 ≤ β < α ⇒ **hyperbola**. Through the vertex you get degenerate cases (point, line, pair of lines).



Standard parabola, ellipse, hyperbola and circle

02 Circle

$$(x-h)^2+(y-k)^2=r^2$$

General: $x^2+y^2+2gx+2fy+c=0 \Rightarrow$ centre $(-g,-f)$, $r=\sqrt{(g^2+f^2-c)}$

Set of points at fixed distance r from the centre. If $g^2 + f^2 - c = 0$, it's a point circle; if < 0, no real circle.

03 Parabola

Set of points equidistant from a fixed point (**focus**) and a fixed line (**directrix**). Eccentricity e = 1.

Equation	Focus	Directrix	Axis	Opens
$y^2 = 4ax$	(a, 0)	x = -a	x-axis	Right
$y^2 = -4ax$	(-a, 0)	x = a	x-axis	Left
$x^2 = 4ay$	(0, a)	y = -a	y-axis	Up
$x^2 = -4ay$	(0, -a)	y = a	y-axis	Down

Latus rectum = 4a (chord through focus, $\perp$ to axis). Vertex at origin.

04 Ellipse

Sum of distances from two foci is constant (= 2a).

$$x^2/a^2 + y^2/b^2 = 1 \ (a > b)$$

$c^2 = a^2 - b^2$, $e = c/a < 1$

Foci (±c, 0), vertices (±a, 0), major axis 2a, minor axis 2b, latus rectum $2b^2/a$. If the larger denominator is under y^2 ($x^2/b^2 + y^2/a^2 = 1$), the major axis is along y: foci (0, ±c). e → 0 gives a circle.

05 Hyperbola

Difference of distances from two foci is constant (= 2a).

$$x^2/a^2 - y^2/b^2 = 1$$

$c^2 = a^2 + b^2$, $e = c/a > 1$

Foci (±c, 0), vertices (±a, 0), transverse axis 2a, conjugate axis 2b, latus rectum $2b^2/a$. For $y^2/a^2 - x^2/b^2 = 1$ the transverse axis is along y. a = b gives a rectangular hyperbola (e = √2).

06 Comparison table

	Parabola $y^2 = 4ax$	Ellipse $x^2/a^2 + y^2/b^2 = 1$	Hyperbola $x^2/a^2 - y^2/b^2 = 1$
e	1	< 1, $\sqrt{(1 - b^2/a^2)}$	> 1, $\sqrt{(1 + b^2/a^2)}$
c	—	$\sqrt{(a^2 - b^2)}$	$\sqrt{(a^2 + b^2)}$
Foci	(a, 0)	(±c, 0)	(±c, 0)
Vertices	(0, 0)	(±a, 0)	(±a, 0)
Latus rectum	4a	$2b^2/a$	$2b^2/a$
Key property	SP = PM	SP + S'P = 2a	$\|SP - S'P\| = 2a$

07 How to identify a conic quickly

- Only one variable squared ⇒ **parabola**. Squared variable = the one perpendicular to the axis (y^2 ⇒ axis along x).
- Both squared, both + with different denominators ⇒ **ellipse**; larger denominator decides the major axis.
- Both squared, one minus ⇒ **hyperbola**; positive term decides the transverse axis.
- Both squared, same coefficient, + ⇒ **circle**.

08 JEE Extra + Tricks

JEE LEVEL

- Tangent conditions: $y = mx + c$ touches parabola $y^2 = 4ax$ if $c = a/m$; ellipse if $c^2 = a^2m^2 + b^2$; hyperbola $c^2 = a^2m^2 - b^2$.
- Parametric points: parabola $(at^2, 2at)$; ellipse $(a\cos\theta, b\sin\theta)$; hyperbola $(a\sec\theta, b\tan\theta)$.
- Latus rectum: parabola $4a$; ellipse and hyperbola $2b^2/a$. Eccentricity: ellipse $e^2 = 1 - b^2/a^2$, hyperbola $e^2 = 1 + b^2/a^2$.
- Focal chord of parabola: $t_1t_2 = -1$; tangents at ends of focal chord are perpendicular and meet on directrix.
- Director circle: ellipse $x^2 + y^2 = a^2 + b^2$; locus of points from which perpendicular tangents are drawn.
- Circle: length of tangent from $(x_1, y_1) = \sqrt{S_1}$; two circles orthogonal if $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

09 Solved examples

Q1. Centre and radius of $x^2 + y^2 - 4x - 8y - 45 = 0$.

$g = -2$, $f = -4 \Rightarrow$ centre $(2, 4)$, $r = \sqrt{(4 + 16 + 45)} = \mathbf{\sqrt{65}}$.

Q2. Focus, directrix, latus rectum of $y^2 = -8x$.

$4a = 8$, $a = 2$, opens left $\Rightarrow$ focus $(-2, 0)$, directrix $x = 2$, LR $= 8$.

Q3. Foci and e of $x^2/25 + y^2/9 = 1$.

$a = 5$, $b = 3$, $c = 4 \Rightarrow$ foci $(\pm4, 0)$, $e = \mathbf{4/5}$.

Q4. Foci and e of $9y^2 - 4x^2 = 36$.

$y^2/4 - x^2/9 = 1 \Rightarrow a = 2$, $b = 3$, $c = \sqrt{13} \Rightarrow$ foci $(0, \pm\sqrt{13})$, $e = \mathbf{\sqrt{13}/2}$.

10 Common mistakes

- Using $c^2 = a^2 - b^2$ for a hyperbola (it's $a^2 + b^2$).
- Taking a as the number under x^2 even when the bigger number is under y^2.
- Latus rectum of parabola as 2a (it's 4a).
- Forgetting to make the RHS 1 before reading a and b.

11 Practice questions

1. Equation of the circle with centre (−3, 2) and radius 4.
2. Equation of the parabola with focus (0, −3) and vertex at origin.
3. Ellipse with vertices (±13, 0) and foci (±5, 0).
4. Hyperbola with foci (±5, 0) and transverse axis 8.

ANSWERS

1) $x^2 + y^2 + 6x - 4y - 3 = 0$ · 2) $x^2 = -12y$ · 3) $x^2/169 + y^2/144 = 1$ · 4) $x^2/16 - y^2/9 = 1$

12 Quick revision: 5 minutes before the exam

CIRCLE

- Centre $(-g, -f)$.
- $r = \sqrt{(g^2 + f^2 - c)}$.

PARABOLA

- $y^2 = 4ax$: S(a,0), x = −a.
- LR = 4a, e = 1.

ELLIPSE

- $c^2 = a^2 - b^2$, $e < 1$.
- LR = $2b^2/a$.

HYPERBOLA

- $c^2 = a^2 + b^2$, $e > 1$.
- LR = $2b^2/a$.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.