FREE NOTES · NCERT + JEE LEVEL

Straight Lines

Slope of a line, parallel and perpendicular lines, angle between lines, collinearity, all forms of the equation of a line, general form, distance of a point from a line and between parallel lines.

01 Basics from Class 10
02 Slope (gradient)
03 Parallel, perpendicular & angle between lines
04 Forms of the equation of a line
05 General equation Ax + By + C = 0
06 Distance formulas
07 Formula sheet
08 JEE Extra + Tricks
09 Solved examples
10 Common mistakes
11 Practice questions
12 Quick revision

01 Basics from Class 10

Distance PQ $= \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

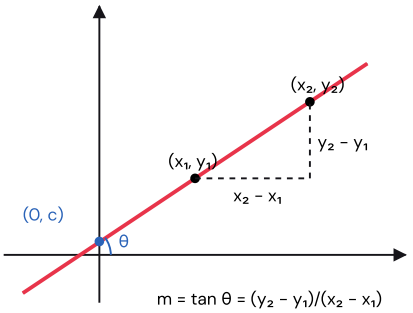
Section (m:n internally) $= ((mx_2 + nx_1)/(m + n), (my_2 + ny_1)/(m + n))$

Midpoint $= ((x_1 + x_2)/2, (y_1 + y_2)/2)$; centroid $= ((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3)$

Area of triangle $= ½ \,|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

02 Slope (gradient)

- m = tan θ, θ = inclination with positive x-axis (0 ≤ θ < π).
- Through two points: m = $(y_2 - y_1)/(x_2 - x_1)$.
- Horizontal line: m = 0. Vertical line: slope not defined.
- m > 0: line rises to the right; m < 0: falls.



Slope from two points; y-intercept c

03 Parallel, perpendicular & angle between lines

Parallel: $m_1 = m_2$

Perpendicular: $m_1\, m_2 = -1$

$$\tan\theta = |(m_2 - m_1)/(1 + m_1 m_2)|$$

Gives the acute angle. Three points A, B, C are **collinear** if slope AB = slope BC.

04 Forms of the equation of a line

Form	Equation
Horizontal / vertical	$y = a;\ x = b$
Point-slope	$y - y_1 = m(x - x_1)$
Two-point	$y - y_1 = [(y_2 - y_1)/(x_2 - x_1)](x - x_1)$
Slope-intercept	$y = mx + c$ (c = y-intercept)
Intercept	$x/a + y/b = 1$
Normal	$x\cos\omega + y\sin\omega = p$

Normal form: p = perpendicular distance from origin, ω = angle the perpendicular makes with the x-axis.

05 General equation Ax + By + C = 0

Slope $= -A/B$, x-intercept $= -C/A$, y-intercept $= -C/B$

Normal form: divide by $\pm\sqrt{(A^2 + B^2)}$ so that the constant becomes negative on the left $(p > 0)$

Line parallel to Ax + By + C = 0: Ax + By + k = 0. Perpendicular: Bx − Ay + k = 0.

06 Distance formulas

$$d = |Ax_1 + By_1 + C| \,/\, \sqrt{(A^2 + B^2)}$$

Distance of (x_1, y_1) from Ax + By + C = 0.

$$d = |C_1 - C_2| \,/\, \sqrt{(A^2 + B^2)}$$

Distance between parallel lines Ax + By + C_1 = 0 and Ax + By + C_2 = 0 (same A, B!).

07 Formula sheet

Slope	$(y_2 - y_1)/(x_2 - x_1) = \tan\theta$
Angle	$\tan\theta = \lvert(m_2 - m_1)/(1 + m_1m_2)\rvert$
Perpendicular	$m_1m_2 = -1$
Point-slope	$y - y_1 = m(x - x_1)$
Intercept	$x/a + y/b = 1$
Point–line distance	$\lvert Ax_1 + By_1 + C\rvert/\sqrt{(A^2 + B^2)}$

08 JEE Extra + Tricks

JEE LEVEL

- Distance between parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$: $|c_1 - c_2|/\sqrt{(a^2 + b^2)}$.
- Angle bisectors: $(a_1x + b_1y + c_1)/\sqrt{(a_1^2 + b_1^2)} = \pm(a_2x + b_2y + c_2)/\sqrt{(a_2^2 + b_2^2)}$.
- Family of lines through intersection: $L_1 + \lambda L_2 = 0$.
- Foot of perpendicular / image of (x_1, y_1) in $ax + by + c = 0$: $(x - x_1)/a = (y - y_1)/b = -k(ax_1 + by_1 + c)/(a^2 + b^2)$, $k = 1$ foot, $k = 2$ image.
- Centroid, incentre $(ax_1 + bx_2 + cx_3)/(a + b + c)$, circumcentre, orthocentre: centroid divides orthocentre–circumcentre line in 2 : 1.
- Area of triangle by determinant $\frac{1}{2}|x_1(y_2 - y_3) + \ldots|$; zero $\Rightarrow$ collinear.

09 Solved examples

Q1. Find the acute angle between lines with slopes 2 and 3.

$\tan\theta = |(3 - 2)/(1 + 6)| = 1/7 \Rightarrow \theta = \tan^{-1}(1/7)$.

Q2. Equation of the line through $(-2, 3)$ with slope -4.

$y - 3 = -4(x + 2) \Rightarrow$ **$4x + y + 5 = 0$**.

Q3. Line through $(1, 2)$ perpendicular to $x - 2y + 3 = 0$.

Slope of given $= \frac{1}{2} \Rightarrow$ required $-2 \Rightarrow y - 2 = -2(x - 1) \Rightarrow$ **$2x + y - 4 = 0$**.

Q4. Distance of $(3, -5)$ from $3x - 4y - 26 = 0$.

$|9 + 20 - 26|/5 =$ **3/5**.

Q5. Distance between $3x - 4y + 7 = 0$ and $6x - 8y + 4 = 0$.

Make A, B equal: $6x - 8y + 4 = 0 \Rightarrow 3x - 4y + 2 = 0 \Rightarrow d = |7 - 2|/5 =$ **1**.

10 Common mistakes

- Using the distance formula for parallel lines without first making A and B equal.
- Saying the slope of a vertical line is 0 (it is undefined).
- Forgetting the modulus in the angle formula (gives obtuse/negative).
- Writing slope of $Ax + By + C = 0$ as A/B (it is $-A/B$).

11 Practice questions

1. Slope of the line through $(3, -2)$ and $(-1, 4)$?
2. Are $(1, 5)$, $(2, 3)$, $(-2, 11)$ collinear?
3. Equation of the line with intercepts 3 and -2.
4. Find k if $kx + 3y = 2$ is perpendicular to $2x - y = 5$.

ANSWERS

1) $-3/2$ · 2) Yes (slopes both -2) · 3) $2x - 3y = 6$ · 4) $k = 3/2$

12 Quick revision: 5 minutes before the exam

SLOPE

- $m = \tan \theta$.
- Parallel equal; $\perp$ product -1.

FORMS

- $y = mx + c$.
- $x/a + y/b = 1$.
- $x \cos \omega + y \sin \omega = p$.

GENERAL

- Slope $-A/B$.
- Parallel: same A, B.

DISTANCE

- $|Ax_1+By_1+C|/\sqrt{(A^2+B^2)}$.
- $|C_1 - C_2|/\sqrt{(A^2+B^2)}$.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.