FREE NOTES · NCERT + JEE LEVEL

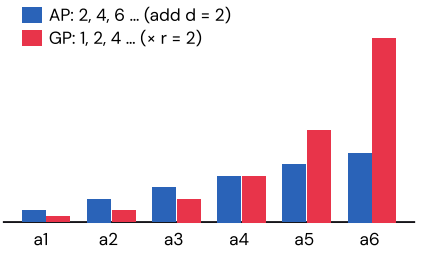
Sequences & Series

Sequences aur series, arithmetic progression (AP) aur arithmetic mean, geometric progression (GP), infinity tak ka jod, geometric mean, AM–GM rishta aur special series Σn, Σn^2, Σn^3 ke jod.

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01 Sequence & series

Sequence ek order wali list $a_1, a_2, a_3, \ldots$ (N pe ek function). **Series** unka jod $a_1 + a_2 + \ldots$ (Σa_k). Finite ya infinite. Fibonacci: $a_1 = a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$.



AP jodne se badhta, GP guna karne se

02 Arithmetic progression (AP)

$a, a + d, a + 2d, \ldots$ (common difference d)

$$a_n = a + (n - 1)d$$

$$S_n = n/2\,[2a + (n - 1)d] = n/2\,(a + l)$$

l = aakhri term. $a_n = S_n - S_{n-1}$. Aakhir se nth term = $l - (n - 1)d$. AP me teen terms: $a - d, a, a + d$.

Properties: constant jodne/ghatane/guna karne se AP bana rehta. a, b, c AP me $\Rightarrow 2b = a + c$.

03 Arithmetic mean

a aur b ka AM $= (a + b)/2$

a aur b ke beech n AMs: $d = (b - a)/(n + 1)$, $A_k = a + kd$

n AMs ka jod = $n \times (a + b)/2$.

04 Geometric progression (GP)

$a, ar, ar^2, \ldots$ (common ratio r)

$$a_n = ar^{n-1}$$

$$S_n = a(r^n - 1)/(r - 1) = a(1 - r^n)/(1 - r),\ r \neq 1$$

$r = 1$ ho to $S_n = na$. GP me teen terms: $a/r, a, ar$. a, b, c GP me $\Rightarrow b^2 = ac$.

05 Infinite GP & geometric mean

$$S_\infty = a/(1 - r),\ |r| < 1$$

a aur b (positive) ka GM $= \sqrt{(ab)}$

a aur b ke beech n GMs: $r = (b/a)^{(1/(n+1))}$

Recurring decimal: $0.3^- = 3/10 + 3/100 + \ldots = (3/10)/(1 - 1/10) = 1/3$.

06 AM–GM rishta

POSITIVE A, B KE LIYE

$A = (a + b)/2 \geq G = \sqrt{(ab)}$, barabar jab $a = b$.

a, b equation $x^2 - 2Ax + G^2 = 0$ ke roots $\Rightarrow a, b = A \pm \sqrt{(A^2 - G^2)}$.

Use: $x + 1/x$ ($x > 0$) ka minimum 2, kyunki $(x + 1/x)/2 \geq \sqrt{1}$.

07 Special series ke jod (JEE ke liye useful)

$\Sigma n = n(n + 1)/2$

$\Sigma n^2 = n(n + 1)(2n + 1)/6$

$\Sigma n^3 = [n(n + 1)/2]^2$

General term T_k k ka polynomial ho to use tod ke ye formulas lagao. Jaise $\Sigma k(k + 1) = \Sigma k^2 + \Sigma k$.

08 JEE Extra + Tricks

JEE LEVEL

- $\Sigma n = n(n+1)/2$, $\Sigma n^2 = n(n+1)(2n+1)/6$, $\Sigma n^3 = [n(n+1)/2]^2$.
- AGP ka jod: $S - rS$ trick; infinite $S_\infty = a/(1-r) + dr/(1-r)^2$.
- Telescoping: $T_n = f(n) - f(n-1)$ likho ya partial fractions $1/(n(n+1)) = 1/n - 1/(n+1)$.
- AP me teen sankhya: $a - d, a, a + d$; GP me: $a/r, a, ar$.
- a, b, c AP, GP, HP me $\Rightarrow b = (a+c)/2$, $\sqrt{(ac)}$, $2ac/(a+c)$; do sankhyaon ke liye $AM \cdot HM = GM^2$.
- a aur b ke beech n AMs: $d = (b-a)/(n+1)$; n GMs: $r = (b/a)^{1/(n+1)}$.

09 Solved examples

Q1. 7 se divisible saare 3-digit numbers ka jod.

105, 112, ..., 994: n = (994 − 105)/7 + 1 = 128 ⇒ S = 128/2 (105 + 994) = **70336**.

Q2. GP ka 5th term 81 aur 2nd term 24. GP nikalo.

$ar^4/ar = r^3 = 81/24 = 27/8 \Rightarrow r = 3/2$, a = 16 ⇒ 16, 24, 36, 54, ...

Q3. Infinity tak jod: 1 − 1/3 + 1/9 − ...

a = 1, r = −1/3 ⇒ S∞ = 1/(4/3) = **3/4**.

Q4. 1×2 + 2×3 + ... + n(n + 1) nikalo.

$\Sigma k^2 + \Sigma k = n(n+1)(2n+1)/6 + n(n+1)/2 =$ **n(n + 1)(n + 2)/3**.

Q5. Do numbers ka AM 10 aur GM 8. Numbers?

a, b = 10 ± √(100 − 64) = 10 ± 6 ⇒ **16 aur 4**.

10 Common mistakes

- |r| ≥ 1 pe S∞ lagana.
- a_n = a + nd likhna (n − 1 hota hai).
- Negative numbers pe AM ≥ GM lagana.
- $\Sigma n^3 = [n(n+1)/2]^2$ me square bhool jaana.

11 Formula sheet & practice

AP	$a_n = a + (n-1)d$; $S_n = n/2[2a + (n-1)d]$
GP	$a_n = ar^{n-1}$; $S_n = a(r^n-1)/(r-1)$
Infinite GP	$a/(1 - r)$
Means	$AM = (a+b)/2$, $GM = \sqrt{ab}$, $AM \geq GM$
Σn, Σn^2, Σn^3	$n(n+1)/2$, $n(n+1)(2n+1)/6$, $[n(n+1)/2]^2$

1. 2, 7, 12, ... ka 20th term?
2. 1, 2, 4, ... ke pehle 10 terms ka jod?
3. 3 aur 19 ke beech 3 AMs daalo.

4. Σ (k = 1 se 10) k^2 nikalo.

ANSWERS

1) 97 · 2) 1023 · 3) 7, 11, 15 · 4) 385

12 Quick revision: exam se 5 minute pehle

AP

- $a_n = a + (n-1)d$.
- $S_n = n/2(a + l)$.
- $2b = a + c$.

GP

- $a_n = ar^{n-1}$.
- $S\infty = a/(1-r)$, $|r|<1$.
- $b^2 = ac$.

MEANS

- AM ≥ GM.
- $a, b = A \pm \sqrt{(A^2 - G^2)}$.

SPECIAL SUMS

- $\Sigma n, \Sigma n^2, \Sigma n^3$.
- T_k ko k ki powers me todo.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.