FREE NOTES · NCERT + JEE LEVEL

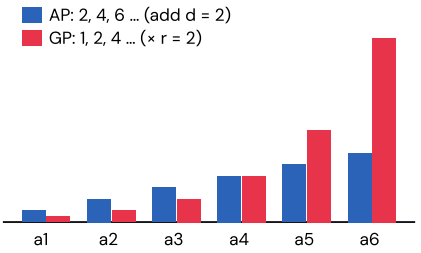
Sequences & Series

Sequences and series, arithmetic progression (AP) and arithmetic mean, geometric progression (GP), sum to infinity, geometric mean, AM–GM relation and sums of special series Σn, Σn^2, Σn^3.

01 Sequence & series
02 Arithmetic progression (AP)
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01 Sequence & series

A **sequence** is an ordered list $a_1, a_2, a_3, \ldots$ (a function on N). A **series** is their sum $a_1 + a_2 + \ldots$ (written Σa_k). Finite or infinite. Fibonacci: $a_1 = a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$.



AP grows by adding, GP by multiplying

02 Arithmetic progression (AP)

$a, a + d, a + 2d, \ldots$ (common difference d)

$$a_n = a + (n - 1)d$$

$$S_n = n/2\,[2a + (n - 1)d] = n/2\,(a + l)$$

l = last term. $a_n = S_n - S_{n-1}$. nth term from the end = l − (n − 1)d. Three terms in AP: a − d, a, a + d.

Properties: adding/subtracting/multiplying by a constant keeps an AP. If a, b, c in AP ⇒ 2b = a + c.

03 Arithmetic mean

AM of a and b $= (a + b)/2$

n AMs between a and b: $d = (b - a)/(n + 1)$, $A_k = a + kd$

Sum of n AMs = n × (a + b)/2.

04 Geometric progression (GP)

$a, ar, ar^2, \ldots$ (common ratio r)

$$a_n = ar^{n-1}$$

$$S_n = a(r^n - 1)/(r - 1) = a(1 - r^n)/(1 - r),\ r \neq 1$$

If r = 1, S_n = na. Three terms in GP: a/r, a, ar. If a, b, c in GP ⇒ b^2 = ac.

05 Infinite GP & geometric mean

$$S\infty = a/(1 - r),\ |r| < 1$$

GM of a and b (positive) $= \sqrt{(ab)}$

n GMs between a and b: r = (b/a)^(1/(n+1))

Recurring decimal: $0.3^{-} = 3/10 + 3/100 + \ldots = (3/10)/(1 - 1/10) = 1/3$.

06 AM–GM relation

FOR POSITIVE A, B

$A = (a + b)/2 \geq G = \sqrt{(ab)}$, equality when $a = b$.

a, b are the roots of $x^2 - 2Ax + G^2 = 0 \Rightarrow a, b = A \pm \sqrt{(A^2 - G^2)}$.

Use: minimum of $x + 1/x$ ($x > 0$) is 2, because $(x + 1/x)/2 \geq \sqrt{1}$.

07 Sums of special series (useful for JEE)

$\Sigma n = n(n + 1)/2$

$\Sigma n^2 = n(n + 1)(2n + 1)/6$

$\Sigma n^3 = [n(n + 1)/2]^2$

For a series with general term T_k = polynomial in k, split it and use these. E.g. $\Sigma k(k + 1) = \Sigma k^2 + \Sigma k$.

08 JEE Extra + Tricks

JEE LEVEL

- $\Sigma n = n(n+1)/2$, $\Sigma n^2 = n(n+1)(2n+1)/6$, $\Sigma n^3 = [n(n+1)/2]^2$.
- AGP (arithmetico-geometric) sum: $S - rS$ trick; infinite $S_\infty = a/(1-r) + dr/(1-r)^2$.
- Telescoping: write $T_n = f(n) - f(n-1)$ or use partial fractions $1/(n(n+1)) = 1/n - 1/(n+1)$.
- Three numbers in AP: $a - d$, a, $a + d$; in GP: a/r, a, ar.
- If a, b, c in AP, GP, HP then $b = (a+c)/2$, $\sqrt{(ac)}$, $2ac/(a+c)$; $AM \cdot HM = GM^2$ (two numbers).
- Insert n AMs between a and b: $d = (b-a)/(n+1)$; n GMs: $r = (b/a)^{1/(n+1)}$.

09 Solved examples

Q1. Sum of all 3-digit numbers divisible by 7.

105, 112, ..., 994: n = (994 − 105)/7 + 1 = 128 ⇒ S = 128/2 (105 + 994) = **70336**.

Q2. The 5th term of a GP is 81 and the 2nd is 24. Find the GP.

$ar^4/ar = r^3 = 81/24 = 27/8 \Rightarrow r = 3/2$, a = 16 ⇒ 16, 24, 36, 54, ...

Q3. Sum to infinity: 1 − 1/3 + 1/9 − ...

a = 1, r = −1/3 ⇒ S∞ = 1/(4/3) = **3/4**.

Q4. Find 1×2 + 2×3 + ... + n(n + 1).

$\Sigma k^2 + \Sigma k = n(n+1)(2n+1)/6 + n(n+1)/2 =$ **n(n + 1)(n + 2)/3**.

Q5. AM and GM of two numbers are 10 and 8. Find them.

a, b = 10 ± √(100 − 64) = 10 ± 6 ⇒ **16 and 4**.

10 Common mistakes

- Using S∞ when |r| ≥ 1.
- Writing $a_n = a + nd$ (it is n − 1).
- Applying AM ≥ GM to negative numbers.
- Forgetting the square in $\Sigma n^3 = [n(n+1)/2]^2$.

11 Formula sheet & practice

AP	$a_n = a + (n-1)d;\ S_n = n/2[2a + (n-1)d]$
GP	$a_n = ar^{n-1};\ S_n = a(r^n-1)/(r-1)$
Infinite GP	$a/(1 - r)$
Means	$AM = (a+b)/2,\ GM = \sqrt{ab},\ AM \geq GM$
$\Sigma n, \Sigma n^2, \Sigma n^3$	$n(n+1)/2,\ n(n+1)(2n+1)/6,\ [n(n+1)/2]^2$

1. 20th term of 2, 7, 12, ...?
2. Sum of first 10 terms of 1, 2, 4, ...?
3. Insert 3 AMs between 3 and 19.

4. Find Σ (k = 1 to 10) k^2.

ANSWERS

1) 97 · 2) 1023 · 3) 7, 11, 15 · 4) 385

12 Quick revision: 5 minutes before the exam

AP

- $a_n = a + (n-1)d$.
- $S_n = n/2(a + l)$.
- $2b = a + c$.

GP

- $a_n = ar^{n-1}$.
- $S\infty = a/(1-r)$, $|r|<1$.
- $b^2 = ac$.

MEANS

- AM ≥ GM.
- $a, b = A \pm \sqrt{(A^2 - G^2)}$.

SPECIAL SUMS

- Σn, Σn^2, Σn^3.
- Split T_k into powers of k.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.