FREE NOTES · NCERT + JEE LEVEL

Binomial Theorem

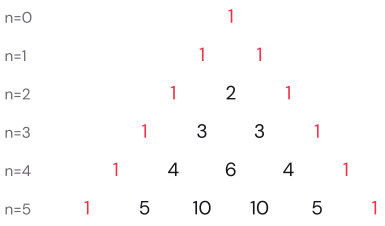
Pascal's triangle, positive integral index ke liye binomial theorem, general term, middle term, x se independent term, binomial coefficients ki properties aur applications.

01 Pascal's triangle
02 Binomial theorem (n positive integer)
03 Special cases
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01 Pascal's triangle

Har number upar ke do numbers ka jod hai. Row n, $(a + b)^n$ ke coefficients deti hai, jo ${}^nC_0$, ${}^nC_1$, ..., ${}^nC_n$ hain.

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ (row 1 3 3 1).



n = 5 tak Pascal's triangle

02 Binomial theorem (n positive integer)

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_n b^n$$

- Terms ki ginti = n + 1.
- a ki power n se 0 tak ghatti, b ki 0 se n tak badhti; har term me powers ka jod = n.
- Dono siron se barabar doori wale coefficients barabar (${}^nC_r = {}^nC_{n-r}$).

03 Special cases

$(a-b)^n = \Sigma\,(-1)^r\,{}^nC_r\,a^{n-r}\,b^r$ (sign badalte rehte)

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$

$x = 1$ rakho: ${}^nC_0 + {}^nC_1 + \ldots + {}^nC_n = 2^n$

$x = -1$ rakho: ${}^nC_0 - {}^nC_1 + {}^nC_2 - \ldots = 0 \Rightarrow$ odd-place ka jod = even-place ka jod $= 2^{n-1}$

04 General term

$$T_{r+1} = {}^nC_r\,a^{n-r}\,b^r$$

(r + 1)th term. (a - b)^n ke liye (-1)^r lagao. Aakhir se rth term = shuru se (n - r + 2)th term.

05 Middle term

n	Middle term(s)
Even	Ek: (n/2 + 1)th term
Odd	Do: ((n + 1)/2)th aur ((n + 3)/2)th terms

Middle term ka binomial coefficient sabse bada hota hai.

06 x se independent term (constant term)

T_{r+1} likho, x ki power jodo, use 0 ke barabar rakho, r nikalo. Example: $(x^2 + 1/x)^9$ me $T_{r+1} = {}^9C_r$ x^(18 - 3r) ⇒ 18 - 3r = 0 ⇒ r = 6 ⇒ $T_7 = {}^9C_6 = 84$.

07 Applications

- **Jaldi powers:** $(98)^5 = (100 - 2)^5 = 9039207968$.
- **Numbers compare:** $1.01^{1000000}$ vs 10000: $(1 + 0.01)^{1000000} > 1 + 1000000 \times 0.01 = 10001 > 10000$.
- **Divisibility:** $6^n - 5n$ ko 25 se bhaag dene pe remainder 1: $6^n = (1 + 5)^n = 1 + 5n + 25(\ldots)$.

08 JEE Extra + Tricks

JEE LEVEL

- General term $T_{r+1} = {}^{n}C_r\, x^{n-r}\, y^{r}$; x se independent term: x ki power = 0.
- Middle term: n even ⇒ (n/2 + 1)th; n odd ⇒ do middle terms.
- Sabse bada coefficient ${}^{n}C_{n/2}$; numerically sabse bada term: $T_{r+1}/T_r \geq 1$ compare.
- Coefficients ka jod: x = 1 rakho; $\Sigma {}^{n}C_r = 2^{n}$; even ka jod = odd ka jod = 2^{n-1}; $\Sigma\, r \cdot {}^{n}C_r = n2^{n-1}$.
- Remainder sawaal: base ko (multiple ± 1)^{n} likho aur expand; aakhri do digits ke liye $(10k \pm 1)^{n}$.

09 Solved examples

Q1. $(2x - 3)^4$ expand karo.

$16x^4 - 96x^3 + 216x^2 - 216x + 81$.

Q2. $(x - 2y)^{12}$ ka 4th term.

$T_4 = {}^{12}C_3\, x^9 (-2y)^3 = 220 \times (-8)\, x^9y^3 =$ **$-1760\, x^9y^3$**.

Q3. $(x + 3)^8$ me x^5 ka coefficient.

$T_{r+1} = {}^8C_r\, x^{8-r}\, 3^r$; $8 - r = 5 \Rightarrow r = 3 \Rightarrow {}^8C_3 \times 27 =$ **1512**.

Q4. $(x/3 + 9y)^{10}$ ka middle term.

$n = 10 \Rightarrow T_6 = {}^{10}C_5\, (x/3)^5\, (9y)^5 = 252 \times 9^5/3^5\, x^5y^5 =$ **$61236\, x^5y^5$**.

10 Common mistakes

- ${}^nC_r$ wale term ko rth term kehna (wo T_{r+1} hai).
- $(a - b)^n$ me b ka sign chhod dena.
- Coefficient nikalte waqt b ka number wala hissa bhool jaana (jaise 3^r).
- Kehna ki n terms hain (n + 1 hote hain).

11 Formula sheet & practice

General term	$T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$
Terms ki ginti	$n + 1$
Coefficients ka jod	2^n
Middle (n even)	$(n/2 + 1)$th
Middle (n odd)	$((n+1)/2)$th, $((n+3)/2)$th

1. $(x + y)^{20}$ me terms?
2. $(1 + x)^{10}$ ke coefficients ka jod?
3. $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$ nikalo.
4. $(x - 1/x)^6$ me x se independent term?

ANSWERS

1) 21 · 2) 1024 · 3) 198 · 4) −20

12 Quick revision: exam se 5 minute pehle

THEOREM

- $(a+b)^n = \Sigma\ {}^nC_r\ a^{n-r}\ b^r$.
- n + 1 terms.

GENERAL TERM

- $T_{r+1} = {}^nC_r\ a^{n-r}\ b^r$.
- Constant term: power = 0.

MIDDLE

- n even: ek.
- n odd: do.

COEFFICIENTS

- $\Sigma = 2^n$.
- Odd jod = even jod = 2^{n-1}.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.