FREE NOTES · NCERT + JEE LEVEL

Binomial Theorem

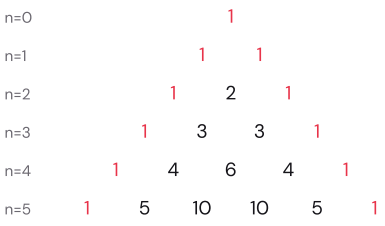
Pascal's triangle, the binomial theorem for positive integral index, general term, middle term, term independent of x, properties of binomial coefficients and applications.

01 Pascal's triangle
02 Binomial theorem (n a positive integer)
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01 Pascal's triangle

Each number is the sum of the two above it. Row n gives the coefficients of $(a + b)^n$, which are ${}^nC_0$, ${}^nC_1$, …, ${}^nC_n$.

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ (row 1 3 3 1).



Pascal's triangle up to n = 5

02 Binomial theorem (n a positive integer)

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_n b^n$$

- Number of terms = n + 1.
- Powers of a go down from n to 0, powers of b go up from 0 to n; sum of powers in each term = n.
- Coefficients equidistant from the ends are equal (${}^nC_r = {}^nC_{n-r}$).

03 Special cases

$(a-b)^n = \Sigma\,(-1)^r\,{}^nC_r\,a^{n-r}\,b^r$ (signs alternate)

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$

Put $x = 1$: ${}^nC_0 + {}^nC_1 + \ldots + {}^nC_n = 2^n$

Put $x = -1$: ${}^nC_0 - {}^nC_1 + {}^nC_2 - \ldots = 0 \Rightarrow$ sum of odd-place = sum of even-place $= 2^{n-1}$

04 General term

$$T_{r+1} = {}^nC_r\,a^{n-r}\,b^r$$

The (r + 1)th term. For $(a - b)^n$ include $(-1)^r$. The rth term from the end = (n − r + 2)th term from the start.

05 Middle term

n	Middle term(s)
Even	One: (n/2 + 1)th term
Odd	Two: ((n + 1)/2)th and ((n + 3)/2)th terms

The middle term has the greatest binomial coefficient.

06 Term independent of x (constant term)

Write T_{r+1}, collect the power of x, set it equal to 0, solve for r. Example: in $(x^2 + 1/x)^9$, $T_{r+1} = {}^9C_r$ x^(18 − 3r) ⇒ 18 − 3r = 0 ⇒ r = 6 ⇒ $T_7 = {}^9C_6 = 84$.

07 Applications

- **Quick powers:** $(98)^5 = (100 - 2)^5 = 9039207968$.
- **Comparing numbers:** $1.01^{1000000}$ vs 10000: $(1 + 0.01)^{1000000} > 1 + 1000000 \times 0.01 = 10001 > 10000$.
- **Divisibility:** $6^n - 5n$ leaves remainder 1 on division by 25: $6^n = (1 + 5)^n = 1 + 5n + 25(\ldots)$.

08 JEE Extra + Tricks

JEE LEVEL

- General term $T_{r+1} = {}^{n}C_r\, x^{n-r}\, y^{r}$; term independent of x: set power of x = 0.
- Middle term: n even ⇒ (n/2 + 1)th; n odd ⇒ two middle terms.
- Greatest coefficient ${}^{n}C_{n/2}$; numerically greatest term: compare $T_{r+1}/T_r \geq 1$.
- Sum of coefficients: put x = 1; $\Sigma {}^{n}C_r = 2^{n}$; sum of even = sum of odd = 2^{n-1}; $\Sigma\, r \cdot {}^{n}C_r = n2^{n-1}$.
- Remainder problems: write base as (multiple ± 1)^{n} and expand; last two digits via $(10k \pm 1)^{n}$.

09 Solved examples

Q1. Expand $(2x - 3)^4$.

$16x^4 - 96x^3 + 216x^2 - 216x + 81$.

Q2. Find the 4th term of $(x - 2y)^{12}$.

$T_4 = {}^{12}C_3\, x^9\, (-2y)^3 = 220 \times (-8)\, x^9 y^3 =$ **$-1760\, x^9 y^3$**.

Q3. Coefficient of x^5 in $(x + 3)^8$.

$T_{r+1} = {}^8C_r\, x^{8-r}\, 3^r$; $8 - r = 5 \Rightarrow r = 3 \Rightarrow {}^8C_3 \times 27 =$ **1512**.

Q4. Middle term of $(x/3 + 9y)^{10}$.

$n = 10 \Rightarrow T_6 = {}^{10}C_5\, (x/3)^5\, (9y)^5 = 252 \times 9^5/3^5\, x^5 y^5 =$ **$61236\, x^5 y^5$**.

10 Common mistakes

- Calling T_r the rth term from ${}^nC_r$ (it is T_{r+1}).
- Dropping the sign of b in $(a - b)^n$.
- Forgetting the numerical part of b while finding a coefficient (e.g. 3^r).
- Saying there are n terms (there are n + 1).

11 Formula sheet & practice

General term	$T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$
Number of terms	$n + 1$
Sum of coefficients	2^n
Middle (n even)	$(n/2 + 1)$th
Middle (n odd)	$((n+1)/2)$th, $((n+3)/2)$th

1. Number of terms in $(x + y)^{20}$?
2. Sum of coefficients of $(1 + x)^{10}$?
3. Find $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$.
4. Term independent of x in $(x - 1/x)^6$?

ANSWERS

1) 21 · 2) 1024 · 3) 198 · 4) −20

12 Quick revision: 5 minutes before the exam

THEOREM

- $(a+b)^n = \Sigma\ {}^nC_r\ a^{n-r}\ b^r$.
- n + 1 terms.

GENERAL TERM

- $T_{r+1} = {}^nC_r\ a^{n-r}\ b^r$.
- Set power = 0 for constant term.

MIDDLE

- n even: one.
- n odd: two.

COEFFICIENTS

- $\Sigma = 2^n$.
- Odd sum = even sum = 2^{n-1}.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.