FREE NOTES · NCERT + JEE LEVEL

Permutations & Combinations

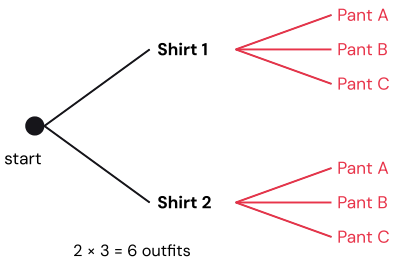
Fundamental principle of counting, factorial, permutations (all distinct, with repetition, with identical objects, circular), combinations, the link between ${}^{n}P_{r}$ and ${}^{n}C_{r}$ and standard counting problems.

01 Fundamental principle of counting
02 Factorial
03 Permutations (arrangements)
04 Special permutations
05 Combinations (selections)
06 Properties of ${}^{n}C_{r}$
07 Selection vs arrangement: how to decide
08 JEE Extra + Tricks
09 Solved examples
10 Common mistakes
11 Formula sheet & practice
12 Quick revision

01 Fundamental principle of counting

- **Multiplication (AND):** if one job can be done in m ways and after it another in n ways, both together in **m × n** ways.
- **Addition (OR):** if two jobs are alternatives (not both), total ways = **m + n**.

Example: 3-letter "words" from ROSE without repetition = 4 × 3 × 2 = 24.



Tree diagram: 2 shirts and 3 pants

02 Factorial

$$n! = 1 \times 2 \times 3 \times \ldots \times n;\ 0! = 1$$

$$n! = n \times (n-1)!$$

5! = 120, 6! = 720, 7! = 5040. 8!/6! = 8 × 7 = 56.

03 Permutations (arrangements)

$$^{n}P_{r} = n!/(n - r)!$$

Arrangements of r objects out of n distinct objects, order matters, no repetition. $^{n}P_{n} = n!$, $^{n}P_{0} = 1$. With repetition allowed: $\mathbf{n^{r}}$.

04 Special permutations

Situation	Number of ways
n objects, p alike of one kind, q of another…	$n!/(p!\ q!\ \ldots)$
Some objects always together	Treat them as one block, then multiply by internal arrangements
Some never together	Total - together
Circular arrangement of n	$(n - 1)!$
Necklace / garland (clockwise = anticlockwise)	$(n - 1)!/2$

Example: arrangements of MISSISSIPPI = 11!/(4! 4! 2!) = 34650.

05 Combinations (selections)

$$^{n}C_{r} = n!/(r!(n - r)!)$$

$^{n}P_{r} = {}^{n}C_{r} \times r!$

Order doesn't matter. Choosing 2 from {A, B, C}: AB, AC, BC = $^{3}C_{2} = 3$ (but $^{3}P_{2} = 6$).

06 Properties of ${}^{n}C_{r}$

${}^{n}C_{r} = {}^{n}C_{n-r}$; ${}^{n}C_{0} = {}^{n}C_{n} = 1$

${}^{n}C_{a} = {}^{n}C_b \Rightarrow a = b$ or $a + b = n$

${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$ (Pascal's rule)

${}^{n}C_{0} + {}^{n}C_{1} + \ldots + {}^{n}C_{n} = 2^{n}$

07 Selection vs arrangement: how to decide

Words in the question	Use
arrange, order, line up, words, numbers, ranks, seating	Permutation
select, choose, committee, team, group, handshakes	Combination

- Diagonals of an n-gon = ${}^{n}C_{2}$ - n. Lines through n points (no 3 collinear) = ${}^{n}C_{2}$. Triangles = ${}^{n}C_{3}$.
- Handshakes among n people = ${}^{n}C_{2}$.

08 JEE Extra + Tricks

JEE LEVEL

- Circular arrangement: $(n - 1)!$; necklace/garland (flip same): $(n - 1)!/2$.
- Distribution of n identical items into r groups (≥ 0 each): ${}^{n+r-1}C_{r-1}$; each ≥ 1: ${}^{n-1}C_{r-1}$.
- Derangements: $D_n = n![1 - 1/1! + 1/2! - \ldots + (-1)^n/n!]$; $D_3 = 2$, $D_4 = 9$.
- Number of divisors of $N = p^a q^b r^c$: $(a + 1)(b + 1)(c + 1)$; sum of divisors product of $(p^{a+1} - 1)/(p - 1)$.
- Rank of a word in dictionary: count words starting with smaller letters step by step.
- Gap method: arrange others first, place restricted items in gaps (no two together).

09 Solved examples

Q1. How many 4-digit numbers can be formed with 1–9 if no digit repeats?

${}^{9}P_{4} = 9 \times 8 \times 7 \times 6 =$ **3024**.

Q2. In how many ways can the letters of INDEPENDENCE be arranged?

12 letters: N 3, E 4, D 2 ⇒ 12!/(3! 4! 2!) = **1663200**.

Q3. A committee of 3 boys and 2 girls from 5 boys and 4 girls?

${}^{5}C_{3} \times {}^{4}C_{2} = 10 \times 6 =$ **60**.

Q4. Words from MOTHER with vowels together?

O, E as one block: 5! × 2! = **240**.

Q5. If ${}^{n}C_{8} = {}^{n}C_{2}$, find ${}^{n}C_{2}$.

n = 10 ⇒ ${}^{10}C_{2} =$ **45**.

10 Common mistakes

- Using P when order doesn't matter (teams, committees).
- Forgetting that a number can't start with 0.
- Forgetting the internal arrangement of a "together" block.
- Using n! instead of (n - 1)! for circular seating.

11 Formula sheet & practice

${}^{n}P_{r}$	$n!/(n-r)!$
${}^{n}C_{r}$	$n!/(r!(n-r)!)$
Repetition allowed	n^{r}
Identical objects	$n!/(p!\,q!\,\ldots)$
Circular	$(n-1)!$
Pascal	${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

1. Number of diagonals of a hexagon?

2. Ways to arrange 5 people around a table?

3. Find r if ${}^{5}P_{r} = 2 \times {}^{6}P_{r-1}$.

4. 3-digit even numbers from 1, 2, 3, 4, 6, 7 (no repetition)?

ANSWERS

1) 9 · 2) 24 · 3) r = 3 · 4) 60

12 Quick revision: 5 minutes before the exam

COUNTING

- AND ⇒ ×, OR ⇒ +.
- 0! = 1.

PERMUTATION

- ${}^{n}P_{r} = n!/(n - r)!$.
- Alike: n!/(p!q!).
- Circle: (n - 1)!.

COMBINATION

- ${}^{n}C_{r} = {}^{n}P_{r}/r!$.
- ${}^{n}C_{r} = {}^{n}C_{n-r}$.

SHORTCUTS

- Diagonals ${}^{n}C_{2} - n$.
- Together = block × inside.
- Never together = total - together.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.